Asset Pricing Inefficiencies: The Key Role Played by Fundamentalists’ Mistakes on Risk Premia and a Faulty Learning Process.

Olivier Davanne

Risk Premium Invest

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Abstract

Three key variables determine the fair value estimates for financial assets: the current and future payoffs provided by the asset, the current and future short-term interest rates controlled by central banks, and the current and future excess returns - or short-term risk premia - required by investors. Although they all contribute to present value calculations in very similar ways, a fundamental difference exists among these variables: while the current returns paid by assets and the short-term interest rates are known, the short-term risk premia remain private information. Despite the availability of some surveys, fundamentalists inevitably make mistakes when estimating this crucial variable.

When these errors are significant and persist over time, a "conundrum" surfaces in the markets. The resulting signals are often challenging to interpret, leading to a loss of confidence in fundamentalist asset pricing and chaotic price overshooting. In this paper, we theoretically discuss this critical mechanism, which, to our knowledge, has not been identified previously. We illustrate how the concept of "fundamentalists’ capitulation" aids in understanding certain enigmatic episodes in the history of US Treasuries and the equities market.

Risk premia change and this is one of the main reasons markets are volatile. However, we argue that such disorderly and costly learning processes are not inevitable. A better understanding of the “market failures” at work (bounded rationality and private information) could help fundamentalists use information about current risk premia more effectively and keep the control of markets.

Bull-markets are born on pessimism, grow on skepticism, mature on optimism and die on euphoria.
Sir John Templeton, Founder of Templeton Mutual Funds

Keywords: Financial assets, fair value, risk premia, fundamentalist asset pricing, market failure, bounded rationality, private information.

Contact: odavanne@riskpremium.com

Introduction

Observed asset prices exhibit many surprising characteristics. But there is one that is more worrying than others.

Sometimes, prices reach levels that are truly surprising. They may be too high, in the sense that the embedded or objective risk premia are too low (i.e., future long-term returns are very likely to be poor) or too low (with extremely high embedded risk premia). Perhaps this significant volatility in embedded risk premia is due to changes in investors' risk aversion throughout the business cycle (see the survey by Cochrane (2017)). But as pointed out in Adam and Nagel (2023) survey of the literature, expectations data seems to contradict that explanation. A key puzzle is that investors often don't seem to spot the abnormal level of risk premia. When prices are too high, surveys show that investors often expect medium to high returns rather than low returns in the short term (typically in the coming year). Short-term risk premia appear to be high, not low. As Greenwood and Schleifer (2014) put it "The evidence is not consistent with rational expectations representative investor models of returns".

But perhaps this apparent inconsistency is just a statistical illusion. It is always difficult to assess in real time the risk premia embedded in asset prices from the perspective of a long-term buy-and-hold investor. So perhaps in hindsight we are now able to say that the risk premia were abnormal, but at the time the information about "objective" risk premia may not have been so clear. This is for example the approach followed by Nagel and Xu (2023). The limitation of this explanation is that it generally relies on weak, and at times extremely weak, proxies of the embedded risk premia. The term "objective" is often a bit misleading. When investors assess embedded risk premia and fair values, they are not limited to the set of indicators used in this literature (dividend yield, slope of the yield curve, interest rates spread in the currency market, etc.). They generally try to use all the available information, including proprietary information based on internal research, to estimate future payoffs, future short-term interest rates, and therefore the embedded long-term risk premia. In other words, most of past "conundrums" are not spotted only now with the benefit of hindsight. They were well spotted in real time after a serious and difficult evaluation of the embedded risk premia (this was for example the case for the extreme overvaluation of the US stock market at the end of the 1990s). Thus, the fundamental puzzle remains: why are the warnings sent out by serious fundamentalist analysts (or central bankers!) sometimes completely ignored by many investors who seem to form irrational expectations?

Perhaps because they (or rather some of them) are naturally irrational… There is indeed a long tradition in part of the literature to blame the responsibility of non-fundamentalist investors who do not base their expectations on established valuation models but tend to extrapolate current trends (see for example Shleifer and Summers (1990), DeLong et al. (1990) and Shiller (2000)). More recently, some works have attempted to make the link between this irrationality and the reasoning bias identified in the behavioral economics literature, in particular by Kahneman and Tversky (1972). As Bordalo et al. (2022)) put it "The expectations of professional forecasters, corporate managers, consumers, and investors appear to be systematically biased in the direction of overreaction to news. As a result, beliefs are too optimistic in good times and too pessimistic in bad times, at the individual level and sometimes at the consensus level as well". They see in this systematic cognitive bias a key explanation of our conundrum since it would influence both the average perception of embedded risk premia (the investors most vulnerable to this bias, would extrapolate recent good cash flows news) and the price expectations (good news on recent capital-gains would lead to bullish expectations).

There is surely something true in these well documented psychological biases. However, this story centered on some irreducible irrationality is not very satisfactory for many reasons:

- There is so much to lose by being an irrational investor. Why don't people learn from their past mistakes? Why do they tend to extrapolate recent trends, despite all the warnings they receive that "past performance is not indicative of future performance"?
- Moreover, rational fundamentalist investors, who base their decisions on present value relations, are not the only ones trying to profit from the mistakes of the irrational trend followers. Among non-fundamentalist investors, there is another key class of investors that seems ignored by this literature (and will play a role in this paper). The clever contrarians who take positions against the crowd. They fight the trend followers and should stabilize markets. Why is the powerful alliance of fundamentalists and contrarians sometimes unable to avoid some serious mispricing?

While there is likely something in the human mind that encourages "trend following" (for both cash flows and prices), that cannot be the entire explanation. There is probably something more fundamental driving markets to lose confidence in rational valuation models and the repeated warnings of fundamentalist economists. In other words, rather than just blaming the trend followers, one should explore the idea that there may also be structural vulnerabilities in how fundamentalist analysts estimate fair values. As Shiller (2013) put it in his Nobel Prize lecture "bubbles are not, in my mind, about craziness of investors. They are rather about how investors are buffeted en masse from one superficially plausible theory about conventional valuation to another. One thinks of how a good debater can take either side of many disputes, and, if the debater on the other side has weak skills, can substantially convince the audience of either side". What are the weak skills of the fundamentalist debaters? Where can fundamentalist investors go wrong in a way that destroys their credibility?

Fair values estimated by "rational" fundamentalist analysts depend on three key variables for each asset class: the future return provided by the asset, the path of short-term rates controlled by central banks and the path of short-term risk premia required by other investors in the future.

A key point is that asset prices depend not only on current risk premia, but also on the short-term risk premia that investors will require in the future (for example, central banks' Quantitative Tightening does not affect current long-term rates by changing investors' current risk aversion, but by changing expected future risk premia as investors will have to gradually absorb a higher supply of long-term government bonds).

Another key point is that there is a fundamental difference in nature between these three variables. Current yields and short-term interest rates are known, and investors need "only" to estimate a credible future path. The current and future short-term risk premia play exactly the same role as the current and future short-term interest rates (they jointly determine how future payoffs should be discounted), but the current short term risk premia are not observable. There is a fundamental problem of missing information. Short-term expected excess returns are private information, unlike current returns and short-term interest rates. It is more difficult to build a likely path when the starting point is missing!

This means that investors can be wrong about their estimate of the fair value. In fact, they are certainly wrong as it is impossible for them to know exactly what risk premia other investors require. For the other two variables, it makes little sense to say that they are wrong. It is true that they don't know the true underlying process driving future cash flows and short rates. But they start from observable values, and it is always possible that they have identified the correct path.

This is a fundamental difference that we believe plays a central role in the instability of financial markets.

Investors need to estimate the current short-term risk premia and build from this starting point the likely future path, and this estimation is in principle extremely challenging. In this estimation process, they can rely on:

- Observable past excess returns on various asset classes over the long-term. This is probably the dominant approach. But the past is not a reliable guide to current short-term risk premia.
- Observable asset prices. We'll revisit this point in greater detail, but it is likely impossible to extract the risk premia expected by investors solely from asset prices, as identical prices can result from markedly different scenarios concerning cash flows and risk premia (in bond market literature, this is referred to as the "hidden factors" problem, see Duffee (2011)).
- The result of surveys. The most direct way to make private information public, and to narrow the fundamental difference in nature between risk premia and other variables, is to ask investors what their expected short-term returns are. But surveys are sometimes very difficult to interpret for the various reasons that we will discuss in this paper.

Thus, this is an essential and highly complex task, with no available guide, academic or professional, outlining the procedure to follow.

In this paper, we argue that in the current situation, i.e., in the absence of guidance on how to proceed, fundamentalist investors, constrained by the availability of data and "bounded rationality", can make big mistakes. They may use risk premia estimates that are far removed from reality for a long period of time. The learning process can be extremely chaotic, and we see this as one of the main reasons for market instability.

The paper is organized as follows:

In the first section, we will discuss from a theoretical perspective the key role played by these short-term unobservable risk premia, their difference in nature with other variables and how the errors of fundamentalists can trigger a chaotic adjustment process and an overshooting of asset prices.

In the second section, we will discuss some empirical evidence from the US Treasuries and US equities markets on the existence of these errors and the role that an inefficient learning process plays in market instability.

In the third section, we will discuss what might happen if the importance of surveys of expected returns was better recognized, with the emergence of better surveys to enrich the work of fundamentalists. Fundamentalists' capitulation and overshooting should disappear, but large irreducible volatility would persist.

This paper is obviously related to the very large and diverse literature on market efficiency and excessive volatility. But it is more directly linked to a narrow but burgeoning part of this literature devoted to the lessons learnt from surveys on investors' expectations.

A recent survey of this literature is provided by Adam and Nagel (2023).

In most papers, one conclusion seems to be that changing risk premia are not responsible for the large changes in observed valuations, as risk premia revealed by surveys do not appear low when markets are expensive (see for example De La O and Myers (2021)). But we show that there may be a dynamic process at work that restores the key role played by the evolution of risk premia in low-frequency changes in valuations. When risk premia are lower than usual (for various reasons related to changing risks or behavioral biases), fundamentalists can miss the structural break. At first, they keep prices at an undervalued level. But they lose investor confidence, and the price starts to rise. There is a vicious circle and fundamentalists may be forced to capitulate. We show that at the end of the capitulation process, prices may have overreacted (embedded risk premia are even lower than where they should be) while investors have become either "passive" or "trend followers" with high return expectations. To paraphrase Robert Shiller, the weak fundamentalist debaters have lost… Thus, the abnormally low initial risk premia, at the beginning of the process, disappear in the surveys as fundamentalists capitulate. For example, we argue that high equity prices in the late 1990s were a clear overshooting but may be related to a structural increase in the demand for equities in the mid-1990s, and an initial reduction, unfortunately not spotted by fundamentalists, in required risk premia.

This complex dynamic process from abnormally low to abnormally high short-term risk premia is exactly what Sir John Templeton described in the famous quote we put on the first page: "Bull-markets are born on pessimism, grow on skepticism, mature on optimism and die on euphoria".

Changing risk premia may be the real underlying cause of many large moves in markets, but due to the private nature of some key information (and bounded rationality), there may be a chaotic adjustment process, and these moves become difficult to pin down ex post to the underlying changes in required risk premia. With better surveys on expectations and a better understanding of the process at work, the future may be different. But changing risk premia will continue to impact markets, albeit in a less chaotic way, and trigger some significant volatility.

1/ How do fundamentalists' mistakes affect asset pricing?

In this theoretical part, we will try to answer some key questions. What prices should emerge in markets dominated by rational fundamentalist investors? Why do they make mistakes and lose control of markets? What are the consequences for asset prices?

The observed price P_t of any financial asset results, as for all prices, from the balance between supply and demand.

This fundamental truth seems to be minimized by the modern asset pricing literature which emphasizes the role of stochastic discount factors (SDF) to price assets. In his preface Cochrane (2005) stated that "rather than use portfolio theory to find a demand curve for assets, which intersected with a supply curve gives prices, we now go to prices directly. One can then find optimal portfolios, but it is a side issue for the asset pricing question". We take here a more traditional route. Asset pricing theories based on the SDF usually have a normative bias: they seek to explain what asset prices should be. Here, we want to revisit some of the observed market failures, and so we need to understand why the demand curve is sometimes struggling to intersect properly with supply[1].

The supply $S_t(P_t$) obviously depends on the price of the asset due to two mechanisms. On the one hand, for many assets, the issuance of new securities depends on the price. For example, private companies can go public if the price is attractive. This is probably a powerful medium-term mechanism, but in the short term, an even more powerful mechanism is that when the price increases, the value of existing securities increases and their share in portfolios increases mechanically. In the short term, if there is a sticky quantity Q_t of securities, the supply is Q_tP_t . We will come back to this simple

[1] Contributing to a discussion on the future of asset pricing, Koijen (2021) also argues that it is important to put the analysis of how supply and demand balance back into the center of asset pricing.

endogenous supply mechanism which obviously plays a very powerful role to balance markets, especially in unhealthy markets not driven by fundamental analysis.

The analysis on demand side is more complex. Throughout the vast literature on portfolio choices, demand is not directly determined by prices, but by expected future returns. In the simple case, known as the "myopic" case, only the expected short-term return R_t plays a role and demand is rebalanced at each period according to the new expected short-term returns. But this "myopic" approach is not optimal: if an asset is judged to be cheap (i.e., its expected long-term return is high), people should be invested, even if short term expected returns are relatively poor. Indeed, in the real life as in the theory of optimal portfolio choice, the demand for risky assets does not only depend on the expected short-term returns (the "tactical" allocation), but also on the expected long-term returns (the "strategic" allocation).

For all assets, there are established theories about what determines long-term equilibrium prices ($E_t(P_{t+H})$ with H large enough). In the long term, barring default, the price of bonds will be their face value (with H the bond maturity). For currencies, PPPs (purchasing power parities) are a kind of anchor (with some adjustments). For equities, the Tobin Q (total value of a firm divided by the value of its tangible and intangible assets) should be close to one (if the structural rents due to imperfect competition are considered intangible assets). Thus, the expected long-term return is equal to $\frac{E_t(P_{t+H})}{P_t}$, plus the future income (dividends, coupons) expected from this asset between t and t+H.

Thus, in a simplified way, considering at this stage all expected future revenues (including short rates controlled by central banks) as exogenous, the price P_t is determined by the basic equation:

$$D_t\left(\frac{P_t}{E_t(P_{t+H})}, R_t\right) = Q_t P_t \quad (1)$$

The equation (1) does not represent a normative theory of price formation. It is a useful simple and positive description of what drives prices, with demand being determined, in a complex time-dependent way, by expected short-term and long-term returns. This equation is indeed fully consistent with the modern asset theory based on the marginal utility of consumption or the stochastic discounted factor[2].

[2] This can be easily demonstrated using the fundamental equation resulting from the consumption-based capital asset pricing model. For any investor active in a specific market i, there should be a strong link between the expected return $E_t(R^i_{t+1})$ and the covariance between this return and his future consumption.

$E_t(R^i_{t+1}) - R^f_t = -\frac{COV_t(u'(C_{t+1}),R^i_{t+1})}{E_t(u'(C_{t+1}))}$ with R^i_{t+1} the asset i return between t and t+1, R^f_t the risk free rate and $u'(C_{t+1})$ the marginal utility of period t+1 consumption (where $u'(C_{t+1})$ is a decreasing function of the consumption C_{t+1}). An investor will demand a high risk premium ($E_t(R^i_{t+1}) - R^f_t$) on assets which offer a low return R^i_{t+1} in bad states of the world (i.e., in situations where $u'(C_{t+1})$ is high). This equation, which can be written more generally with the SDF rather than the marginal utility of consumption, is one of the most important relations in the asset pricing literature (see for example equation 1.13 page 14 in Cochrane (2005)).
What determines the key parameter $COV_t(u'(C_{t+1}), R^i_{t+1})$? The first element is obviously how the investor choses to be exposed to this asset. Thus, this key equation establishes a direct link between the expected short-term returns and the demand for the assets. But expected long-term returns also play an important role. If long-term risk premia are high, the investor will expect to remain highly exposed in the future (periods t+1, t+2….). Then, a low return between t and t+1 could be less penalizing. If this low short-term return comes from a rising risk premium, it mechanically means that future expected returns will improve, which will benefit investors (as long as they expect to stay invested). This is why long-term expected returns also play a key role in $COV_t(u'(C_{t+1}), R^i_{t+1})$. Finally, this key equation establishes a strong (and complex) link between three variables:

The short-term return expected $R_t$is obviously linked to the expectations regarding the future price P_{t+1}.

$$R_t = \frac{E_t(P_{t+1}+C_{t+1})}{P_t} - 1 \text{ (1bis)}$$

Where $E_t(C_{t+1})$ is the expected revenue at date t+1.

Thus, the current price is the solution of:

$$D_t\left(\frac{P_t}{E_t(P_{t+H})}, \frac{E_t(P_{t+1}+C_{t+1})}{P_t} - 1\right) = \ Q_t P_t \quad \text{(1ter)}$$

In a fully rational equilibrium, this equation would be solved recursively by fundamentalist investors, starting from the long-term equilibrium prices $E_t(P_{t+H})$, to extract the future return R_t and the price P_t that are likely to emerge from these equations for all date s>t. In this fully rational equilibrium, prices and returns would follow the expected scenario, except when incoming information triggers changes in the key parameters (the future expected payoffs, the future expected supply or the future attitude relative to risk impacting the demand curve D_t).

A key question is to understand how practically rational fundamentalists in the real world try to solve this incredibly complex system to extract the key parameter R_t that they will use in their own investment decisions.

They have three main difficulties, and they know them:

- The demand side is very uncertain. No one has a good knowledge of the function D_t. Indeed, Koijen (2021) argues that research on asset pricing should give a high priority to a better understanding of the "demand system".
- To make matters worse, every fundamentalist knows that the market is obviously not driven by their own personal expectations (so called first-order beliefs), but by the "representative" investor expectations[3]. To solve this equation, it is necessary to estimate the current "market" view on all the key current parameters (supply, payoffs, investors preferences that play on D_t) and how these expectations are likely to change in the future (if the fundamentalist that is doing this work believes that the current consensus is wrong).
- Finally, the fundamentalist knows that not all investors are rational investors. So, the fully rational equilibrium based on the recursive solution of (1ter) is not what is going to happen. In the true underlying model, one needs to introduce some behavioral bias (for example, maybe a human tendency to extrapolate the recent returns in the future).

Adam and Marcet (2011) noted that "the rational expectations hypothesis (REH) places enormous demands on agents' knowledge about how the market works" and argued that in such situation, it is rational to use information on past returns to forecast future returns. Adam et al. (2017) applied this insight that "with imperfect information about the price process, optimal behavior prescribes that agents use past capital gain observations to learn about the stochastic process governing the behavior of capital gains" to explain long-term data on US stock prices. But despite the enormous difficulties to solve (1ter), true fundamentalists that don't extrapolate prices fortunately exist. They estimate the

the current demand for the risky asset i, its expected excess return in the short term and its expected excess return in the long term as in equation (1).

[3] We do not discuss here who the "representative" investors really are and how to aggregate heterogenous investors with varying wealth and investment constraints (for example on leverage positions).

assets' fair value, and they communicate (and disagree) on these estimates, rating markets as "undervalued", "fairly valued" or "overvalued".

What are the methods that these true fundamentalists use to make their fair value estimates, and what implications do these estimates have on their short-term expected returns? Despite the importance of this (empirical) question, there is no clear answer in the literature. Yet, the process followed by fundamentalists seems relatively clear. To estimate the fair value P_t^* they use the abundant literature on present-value relations that we'll revisit, and they deal with the three difficulties that we have mentioned in the following way:

- They use their own opinions on the key variables that determine fair value. In other words, in a first stage, they introduce into equation (1ter) for all future date s, their own estimates of future payoffs and investors' preferences (so-called first order beliefs).
- They concentrate all the uncertainties concerning the supply and demand functions (now and in the future) in one key parameter: the short-term risk premium required by investors (now and in the future). In other words, the solution of equation (1) can be written $R_t = r_t + \pi_t^f$, where r_t is the short-term risk-free rate and π_t^f the short-term risk premium considered as normal by fundamentalists. Rather than explicitly solving this equation (1), fundamentalists make direct assumptions on the required risk premia (now and in the future).

Thus, they have a much simpler system of two equations for extracting the current fair value based on their own expectations of future pay offs and risk premia.

$R_t = r_t + \pi_t^f$ and $R_t = \frac{E_t(P_{t+1}+C_{t+1})}{P_t} - 1$

These recursive equations lead to an estimated fair value estimate P_t^* which depends on expected future pay offs and risk premia (P_t^* is specific to each fundamentalist, but there is a sort of average that represents what is called the "consensus"). Fundamentalists expect prices to converge towards their own estimate in the future (the current "undervaluation" or "overvaluation" being due to either differences of views with other fundamentalists or the temporary destabilizing impact of non-fundamentalist investors).

In other words, once fair value has been estimated, the short-term return expected by fundamentalists will be $R_t = r_t + \pi_t^f + +F_t\left(\frac{P_t^*}{P_t}\right)$ (2) with $F_t(1) = 0$

When the current price is not aligned with the estimated fundamental price, ($\frac{P_t^*}{P_t} \neq 1$), the speed of the expected return will depend on how quickly fundamentalists believe that other investors will correct their errors. Moreover, the weight given to these fair value estimates in their expected short-term returns depends on their confidence in the accuracy of the model they use. Here the key parameter is $F_t'(1)$. When $F_t'(1)$ is very high, the short term expected return is highly sensitive to any apparent mispricing (i.e., a difference between current prices and fundamental prices).

Finally, we can mix (1) and (2) to achieve our goal of establishing how asset prices are determined in a market controlled by fundamentalist investors:

$$D_t\left(\frac{P_t}{E_t(P_{t+H})}, r_t + \pi_t^f + +F_t\left(\frac{P_t^*}{P_t}\right)\right) = Q_t P_t \quad (3)$$

With P_t^* and π_t^f the "consensus" about fair value and the normal short-term risk premium. Again, this is not a normative asset pricing theory: we believe that equation (3) is a fair representation of the demand coming from fundamentalist investors. This demand depends on both their estimates of the normal current risk premium π_t^f and on their opinion on the current valuation of the market $\frac{P_t^*}{P_t}$. Equation (3) is both simple and very useful to understand the key mechanisms at work.

To finish the job, we need to be a bit more specific about P_t^*, the fair value estimated by the fundamentalists using present-value relations. These present-value relations are not easy to establish in discrete times due to non-linearities. But the formalization in continuous times makes it possible to obtain simple analytical solutions without any loss of economic substance. The formulas that we will establish are not exactly the present value relations used by professional investors and analysts as they use simplified versions, often expressed in discreet time. Or they may even make some implicit mental calculations without the help of a formal model. But we believe that these equations correctly represent the general rational approach - explicit or implicit – that fundamentalists use (or should use) to assess fair values.

Let's call c_t the current yield expressed in continuous time (the pay-off during the small period of time dt is $c_t \; P_t$ dt).

Present-value relations estimate what should be the price today of an asset which is likely to be worth $E_t(P_{t+H})$ at the relevant long-term horizon (depending on the type of asset) and that is likely to provide a revenue $E_t(c_s)$ $(t \leq s \leq t+H)$.

Suppose the asset price follows a simple diffusion model.

$$\frac{dP_t}{P_t} = \mu_t(.)\, dt + \sigma_t(.)\, dz_t$$

With the price's drift - $\mu_t(.)$ - and its volatility - $\sigma_t(.)$ - depending on the state of the economy.

If the market is efficient, i.e., if the price remains on the fundamental path, investors should expect a total return equal to the short-term risk-free rate ($r_t(.)$), plus the short-term risk premium required by investors as estimated by the fundamentalists ($\pi_t^f(.)$). Thus, the drift in the fundamental asset price should be equal to $r_t(.) + \pi_t^f(.) - c_t(.)$.

$$\frac{dP_t^*}{P_t} = (r_t(.) + \pi_t^f(.) - c_t(.))\, dt + \sigma_t(.)\, dz_t \quad (4)$$

By applying the Itô's lemma to equation (4), we obtain the equation followed by the logarithm of the asset's fundamental price:

$$dLog(P_t^*) = \left(r_t(.) + \pi_t^f(.) - c_t(.) - \frac{1}{2}\sigma_t(.)^{\,2}\right) dt + \sigma_t(.)\, dz_t$$

Thus, for any date T in the future:

$$Log(P_T^*) - Log(P_t^*) = \int_t^T (r_s(.) + \pi_s^f(.) - c_s(.) - \frac{1}{2}\sigma_s(.)^{\,2})\; ds + \int_t^T \sigma_s(.)\, dz_s$$

This equation can be used to establish how the current fundamental price depends on the fundamental price expected at any date in the future.

$$E_t\big(Log(P_T^*)\big) - Log(P_t^*) = E_t\left(\int_t^T (r_s(.) + \pi_s^f(.) - c_s(.) - \frac{1}{2}\sigma_s(.)^{\,2})\; ds\right)$$

Or

$$P_t^* = \exp\left(E_t\left(Log(P_T^*)\right)\right) e^{-\int_t^T E_s\,(r_t(.)+\,\pi_s^f(.)-\,c_s(.)-\frac{1}{2}\sigma_s(.)^{\,2})\,ds}$$

Fundamentalist analysts apply this key relation to the long-term horizon t+H where economic theory provides a good basis for forecasting the equilibrium price of assets (depending on the nature of the asset).

$$P_t^* = \exp\left(E_t\left(Log(P_{t+H})\right)\right) e^{-\int_t^{t+H} E_t\,(r_s(.)+\,\pi_s^f(.)-\,c_t(.)-\frac{1}{2}\sigma_s(.)^{\,2})\,ds}$$

Suppose that the expected distribution of future prices at the horizon H is log-normal, with variance $\sigma_{t,H}^2$. Thus, thanks to the properties of the log-normal distribution:

$$E_t\left(Log(P_{t+H})\right) = Log\left(E_t(P_{t+H})\right) - \frac{1}{2}\sigma_{t,H}^2$$

Finally, we get the fundamental pricing equation, accurate for all assets:

$$P_t^* = E_t(P_{t+H})\ e^{\int_t^{t+H} E_t(c_s)ds}\ \ e^{-\int_t^{t+H} E_t(r_s)ds}\ \ e^{-\int_t^{t+H} E_t(\pi_s^f)ds}\ \ e^{\int_t^{t+H}\frac{1}{2}E_t(\sigma_s^2)\ ds\,-\frac{1}{2}\sigma_{t,H}^2} \quad (5)$$

Equation (5) should be true for any horizon H, but it is most useful – in an explicit or implicit form - with a long-term horizon where economic theory provides good insights into what drives the equilibrium prices.

The current fair value P_t^* is the product of five key terms:

$E_t(P_{t+H})$: The "final price", or the expected price of the asset at the long-term horizon H. As we have already said, there is for each type of asset a natural long-term horizon at which one can rely on a well-established economic theory to estimate $E_t(P_{t+H})$. In this regard, the easiest assets are obviously government bonds (when they are considered risk-free) as bonds of maturity H have a fixed redemption value ($\sigma_{t,H}^2$=0). For currencies, at a long-term horizon (H between 5 and 10 years?), there are powerful forces that constrain where exchange rates could be. A country cannot function forever with an overvalued or undervalued exchange rate. The equilibrium exchange rate is the subject of a vast academic literature dealing, for example, the role of the PPPs. For stocks too, there are some powerful competitive forces that constrain stock prices over the long term. At a long horizon (10 or 20 years?), the value of companies should be equal to their revalued net assets (tangible and intangible). Obviously, the difficulty is to estimate the long-term expected value of intangible assets, which depends a lot on how competitive markets are.

$e^{\int_t^{t+H} E_t(c_s)ds}$: obviously, the current price of any asset depends on the return it is likely to provide between now and the chosen long-term horizon H. For equities, $E_t(c_s)$ is simply the expected dividend yield at date s. For currencies, the holder of foreign currencies will receive the foreign interest rates (r_s^* in continuous time): $e^{\int_t^{t+H} E_t(c_s)ds} = e^{\int_t^{t+H} E_t(r_s^*)ds}$. Finally, this term is particularly easy to estimate for a particular type of bonds: zero-coupon bonds that pay no coupons but only have a final redemption value. For them, the term $e^{\int_t^{t+H} E_t(c_s)ds}$ is equal to 0.

$e^{-\int_t^{t+H} E_t(r_s)ds}$: this is simply the actualization term. The current prices should be low when short-term rates controlled by central banks are expected to be high on average in the future.

$e^{-\int_t^{t+H} E_t(\pi_s^f)ds}$: this is the risk premium component of the actualization term, specific to each asset. Current prices should be low if the risk premia required by investors are expected to be high on average in the future.

$e^{\int_t^{t+H}\frac{1}{2}E_t(\sigma_s^2)\ ds\ -\frac{1}{2}\sigma_{t,H}^2}$: this last term is needed because of the slightly complicated relationship between expected long-term returns and expected short-term returns. Let's illustrate the problem in a simple two-period setting, where x_1 is the period 1 return and x_2 the period 2 return.

$$E\big((1+x_1)(1+x_2)\big) = 1 + E(x_1) + E(x_2) + E(x_1\,x_2) = (1+E(x_1))\big(1+E(x_2)\big) + Covar(x_1,x_2)$$

In efficient markets, the covariance between returns at different dates should be negative. As already noted, when interest rates or risk premia increase, the price of assets declines, but expected future returns rise. Thus, the expected long-term return ($E\big((1+x_1)(1+x_2)\big)$) is lower than the geometrical average of future expected return ($(1+E(x_1))\big(1+E(x_2)\big)$)[4]. The term $e^{\int_t^{t+H}\frac{1}{2}E_t(\sigma_s^2)\ ds\ -\frac{1}{2}\sigma_{t,H}^2}$ is always positive[5] and should boost the current price of the risky asset (a lower long-term risk premium means a higher price).

Now, with equation (3) and (5), we have the complete system to understand how assets prices are determined in financial markets controlled by fundamentalists who form expectations on the long-term equilibrium prices ($E_t\big(P_{t+H}\big)$), the future returns provided by the assets ($E_t(c_s)$), the future path of monetary policy ($E_t(r_s)$) and, last but not least, the current π_t^f and future ($E_t(\pi_s^f)$) risk premia required by investors[6].

As we have already said, fundamentalist investors obviously do not elaborate such explicit scenarios in continuous time and do not exactly use the key present value relations based on equation (5). They can use some simplified discrete time versions[7]. Or they may have a simplified mental process to assess fair values. We can use the analogy of billiards in this respect: a good billiard player does not solve in real time the complex mathematical equation which governs the movement of the ball, but the billions of neurons in his brain are able to provide a simulation of great quality. The same applies to fundamentalist investors: they understand the key role played by current and future payoffs, current and future short-term rates and current and future risk premia (if not, they are not fundamentalists…).

This pragmatic process to solve (1ter) seems quite rational and should lead to prices that are easy to understand as they should reflect average expectations about future pay offs (some fundamentalist having lower expectations are underweight while others are overweight) and reasonable estimates of

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[4] An interesting consequence is that risk premia cannot equal 0 for all horizons. If all short-term risk premia (π_s) are equal to 0, there will be a negative risk premium for long-term buy and hold investors.

[5] $\int_t^{t+H} E_t(\sigma_s^2)\ \ ds$ is the conditional variance of the asset price at the horizon H due mechanically to all the shocks between t and t +H. But, as explained, the shocks on short-term rates and risk premia are partially or fully offset in the long term. Thus, the true conditional variance ($\sigma_{t,H}^2$) is lower. The extreme case is that of risk-free bonds where the final redemption value is perfectly known and $\sigma_{t,H}^2$=0.

[6] Here again, the investors are heterogenous and do not have the same expectations. Here, we implicitly consider the average expectations of all investors. The complex discussion about who the marginal investors are is not very useful at this stage.

[7] For stocks, Campbell and Shiller (1988) proposed an approximation of the fundamentalists' present value relation in discreet time. The iterative logic is exactly the one we described in continuous time, but ultimately the role of future expected short-term rates and risk premia is slightly different from what appears in equation (5). In equation (5), we have a simple integral: all expected future discount rates have the same weight. In the Campbell and Shiller approximation, the weight given to expected discount rates far into the future declines. The reason is that they introduce dividends expressed in level whereas in equation (5), to make the equation tractable we work with a yield (dividend/price). If future expected discount rates increase at date s >t and the level of expected dividends remain unchanged, yields in equation (5) will improve between date t and s (same D and lower P). Thus, the direct impact of the higher expected discount rate will be lower.

future risk premia. It is difficult to understand how this system could produce the pricing inefficiencies that we observe.

But the key point is that this system is not robust. Two vulnerabilities can weaken the control exercised by rational fundamentalists over the markets. The first one, very well known, is that there may be little incentive to be an active fundamentalist. The second one, central to this paper, is that fundamentalists are prone to making mistakes.

The lack of incentives to be an active fundamentalist is the key insight provided by Grossman and Stiglitz (1981). The alternative is to be a passive rational investor. Estimating P_t^* is costly because the active fundamentalist must form expectations on complex variables $(E_t(P_{t+H}), E_t(c_s), E_t(r_s), E_t(\pi_s^f))$. Thus, all investors are faced with the same question: how much should I allocate to my research activities to gain insight into the fair value P_t^* and improve my asset allocation? If a lot of people are doing this research, the current price should not be far from fair value and it may make sense to stop studying P_t^* and just wait for the "normal" excess returns, i.e. expect $R_t = r_t + \pi_t^f$. Prices will then diverge from P_t^* by lack of active arbitrage (more on that later) and this may reintroduce some apparent incentives to be an active fundamentalist. But as Grossman and Stiglitz (1981) pointed out, at the end of this complex game, markets cannot be fully informationally efficient ($P_t \neq P_t^*$) in order to keep some incentives to conduct research, and there will be some instability as the share of active and passive investors varies over time[8].

The second key vulnerability of the asset pricing process that we have described is that fundamentalists are prone to making mistakes. Thus, they can lose control of the market.

To discuss this key point, we must first define what we mean by "mistakes". As the old saying goes, "It's hard to make predictions, especially about the future" (attributed to Mark Twain and a whole host of other people). Thus, fundamentalists' set of expectations $(E_t(P_{t+H}), E_t(c_s), E_t(r_s), E_t(\pi_s^f))$ obviously does not predict exactly the future. There will be surprises and expectations and prices will adjust in the future to these surprises. We cannot really speak here of mistakes[9] and there is no obvious reason to believe that these surprises may destroy the pricing process that we have just described.

But there is one key variable – and only one - where we can really use the word mistake: the normal short-term risk premium π_t^f that enters fundamentalists' estimates (but also determines the demand from rational passive investors). Active fundamentalists assume that their estimate of the normal risk premium π_t^f is the risk premium that would balance demand and supply at the fair value P_t^*.

[8] To properly describe this complex game, one needs to introduce some irrational investors who are active, but with a flawed model (for example as pure trend followers). They contribute to the pricing inefficiencies, but they keep some incentives for conducting research. For recent references to models where rational informed traders (i.e., active fundamentalists) trade with rational uninformed traders (i.e. rational passive fundamentalists) and irrational "noise traders", see Papadimitriou (2023).

[9] However, with the benefit of hindsight, many papers show that some of these surprises might have been expected (see for example Cieslak (2018) and Piazzesi et al. (2015) for errors regarding the future path of short-term rates). Investors do not seem to be using all available information efficiently. There are two possible explanations. First, fundamentalists may produce a superficial analysis of future key variables. This may be due to various biased incentives in the asset management industry that do not push for a thorough (and costly) analysis of the long-term future. Secondly, this observation of some form of basic irrationality can also result from excessive use of the "benefit of hindsight". The statistical relations that are now apparent were perhaps not so clear with only "in-sample" information (see Nagel and Xu (2023)).

Thus, they assume that $D_t\left(\frac{P_t^*}{E_t(P_{t+H})}, r_t + \pi_t^f\right) = Q_t P_t^*$. But obviously, that cannot be exactly the case. There is a true underlying risk premium π_t^* dependent on P_t^* that would clear the market at this specific price ($D_t\left(\frac{P_t^*}{E_t(P_{t+H})}, r_t + \pi_t^*\right) = Q_t P_t^*$) but active fundamentalists have no ways of measuring it accurately. They do not know the demand function, and therefore cannot solve this equation analytically.

They may try to use surveys to measure the true current risk premium π_t , the difference between the average return expected by investors in the short term and the short-term risk-free rate. But these surveys can be tricky to use (much more on that later). Moreover, the true current risk premium can be far from the normal risk premium π_t^* when prices are not at fair value as $D_t\left(\frac{P_t}{E_t(P_{t+H})}, r_t + \pi_t\right) = Q_t P_t$. For example, if all investors are passive, the current risk premium will be π_t^f as this is how passive investors forecast the short-term returns. Surveys will send a green light to active fundamentalists (the risk premium they use in their fair value estimates seems realistic), but this would be a false signal.

Thus, due to private information (i.e., the limitations of surveys) and the complex determinants of short-term risk premia, fundamentalist investors can be wrong about the current risk premium that would clear the market at fair value (i.e. $\pi_t^f \neq \pi_t^*$). In fact, they are always wrong, the only question is by how much. And obviously, if π_t^f is false, $E_t(\pi_t^f)$ for s close to t will also be incorrect: fundamentalists could form expectations about future risk premia that have little to no chance of being accurate. Thus, we can really employ the word "mistakes" and that makes risk premia very different from other parameters that go into fundamentalists' estimates. The current yield (c_t) and short-term interest rates (r_t) are observable. So, expectations for these variables in the short-term future may be a bit naïve, but they cannot be completely wrong because there is always a chance that the economy behaves in the way that fundamentalists expect.

Thus, the question is what happens to asset prices if fundamentalists make mistakes about current risk premia? The answer is clear: several "conundrum" will appear in the behavior of the market.

First, prices will not be exactly at the level expected by active fundamentalists. To simply describe the mechanisms at work, suppose that a proportion α of investors are active fundamentalists and (1- α) passive fundamentalists (no irrational noise traders at this stage). Passive fundamentalists expect a short-term excess return π_t^f and active fundamentalists a short-term excess return $\pi_t^f + F_t\left(\frac{P_t^*}{P_t}\right)$.

Thus, without discussing complex aggregation issues, the average expected excess return (maybe measured by surveys) will be $\pi_t = \alpha\left(\pi_t^f + F_t\left(\frac{P_t^*}{P_t}\right)\right) + (1-\alpha)\,\pi_t^f$.

$$\pi_t = \pi_t^f + \alpha\, F_t\left(\frac{P_t^*}{P_t}\right)$$

In other words, in a market entirely controlled by fundamentalists (active and passive), there is a simple relation between the fundamentalists' apparent (average) mistake which can be (imperfectly) revealed by surveys (π_t- π_t^f) and the puzzling divergence between observed and estimated prices.

$$P_t = \frac{P_t^*}{F_t^{-1}\left(\frac{\pi_t - \pi_t^f}{\alpha}\right)} \quad (6)$$

This fundamental equation shows that in the current price P_t , two different risk premia play a role. The future short-term risk premia estimated by active fundamentalists $E_t(\pi_s^f)$ which determine the estimated fair value P_t^*, and the true current short-term risk premium if it is different from fundamentalists assumption, that determines how the current price is positioned relative to P_t^*.

If the true current risk premium is lower than what fundamentalists – active and passive - consider normal, the price will be higher than its fair estimate (beware however that this relation is only true when fundamentalists control the market: with irrational traders, equation (6) does not hold). However, the difference can be quite small as long as active fundamentalists are willing to take large positions when assets are deemed mispriced. In other words, as long as $F_t'(1)$ is large, P_t cannot be far from P_t^* as estimated by active fundamentalists. A low risk premium not spotted by fundamentalists means that the underlying demand for the asset is stronger than they think. But, as prices rise, active fundamentalists will limit the upside by (mistakenly) going short in the market.

Mistakes made on risk premia by fundamentalists will have two other key consequences:

First, the true fundamental value will be different from what fundamentalists expect. P_t^* is a biased estimate of the true fundamental value. The reason is that if π_t^f is badly estimated, as we have already said, expectations on future risk premia also risk being biased. As the current fair price depends on $e^{-\int_t^{t+H} E_t(\pi_t^f)ds}$, the error can be quite large. If probable risk premia are overestimated by 1% in the next ten years, the equilibrium price will be 10% higher than the fundamentalists' estimate.

Second, the ex-post return will surprise active fundamentalists who will not realize the expected profits on their short or long positions. For a stabilized apparent error (i.e., $\pi_t - \pi_t^f$ broadly constant), the difference between the observed prices and (wrongly) estimated fundamental prices will be stable. That means that they will not get the abnormal return ($r_t + \pi_t^f + F_t\left(\frac{P_t^*}{P_t}\right)$) that they expect as a reward for their short or long arbitrage position.

If there is only a temporary disturbance in risk premia and the error made by fundamentalists does not last long, the various signals pointing to a mispricing will be weak and difficult to spot. Their estimated fundamental price will be not very far from the true fundamental value and the observed prices and returns will only be temporarily confusing.

But more structural and lasting errors will have far-reaching consequences. At some point active fundamentalists will necessarily realize that there is something wrong in the way markets behave. But they cannot be sure of the reasons.

They may hesitate between two broad categories of potential causes:

- Some irrational traders may have a disproportionate impact on prices. Irrationality can take different forms. The radical one is to be a pure trend follower and not to give any weight to estimated fair values. A less radical one is to use present value relations with naïve inputs about future pay offs (for example extrapolating the recently observed trend in corporate profits far into the future).
- The active fundamentalists' models may be flawed. The errors may relate to future cashflows or the fundamentalists may have missed something fundamental in the strength of the demand for the asset under consideration. In other words, estimates of π_t^f may be wrong.

Fundamentalists (active and rational passive) have no way of being sure of the underlying causes that create markets' puzzling behavior. Surveys on expectations should be one of the key sources of information, despite their limitations. But they do not directly give the risk premia that should be used in present-value relations. They are also impacted by the market dysfunctions (for example, as already explained, if investors are mainly passive fundamentalists, there will be no information in the surveys: the apparent risk premia will be exactly the risk premia expected by fundamentalists!). Surveys are very useful canaries in the coal mine: they help to identify the existence of mispricing, but they do not precisely measure the extent of this mispricing.

What are active fundamentalists doing in this kind of situation with the current state of knowledge? Are they able to spot the structural (or quasi-structural) change in underlying risk premia and keep the control of the market?

This is an empirical question that has no obvious answer. The difficulty of properly analyzing in real time why prices are behaving strangely is well illustrated by the famous "conundrum" speech given by Alan Greenspan in 2005. As we will see in the next section, there was an unspotted break in risk premia on US Treasuries in the early 2000s and the market began to behave confusingly. Alan Greenspan noted: "For the moment, the broadly unanticipated behavior of world bond markets remains a conundrum. Bond price movements may be a short-term aberration, but it will be some time before we are able to better judge the forces underlying recent experience"[10].

Despite all his experience and the full support of Fed staff, Alan Greenspan could not precisely explain the origin of the conundrum. Indeed, we'll show in the next section that fundamentalist analysts did not change at that time their estimates on what they believe to be the normal risk premia π_t^f.

Rather than correcting their mistakes, fundamentalists who don't understand what is going on tend to capitulate (maybe the future will be different…) and give less weight to their fundamental estimates. So, to keep it simple, we assume that they more easily change the speed at which they see prices going to fair value (i.e., they lower $F_t'(1)$) rather than changing P_t^*[11]. Obviously, this means that prices will deviate even further from their estimated fundamental level, which will trigger a vicious circle with even more capitulations. It can also be observed that in this process of cumulative capitulation, a trend is likely to appear as prices gradually deviate from (poorly) estimated fundamentals. This trend will be played by chartists and other (not so irrational!) momentum investors who will speed up the process, which risks becoming very chaotic. And in this process, surveys on expectations will lose even more of their informative value.

Where could prices finally converge at the end of the process? Difficult to say with increasing weight given to non-fundamental investment techniques (chartist, contrarian…), but for active fundamentalists the end of the road is $F_t\left(\frac{P_t^*}{P_t}\right) = 0$. They will stop giving any weight to their estimate of the fundamental price and become passive fundamentalists: their expected short-term return is

[10] February 17, 2005, testimony before the Committee on Banking, Housing, and Urban Affairs of the U.S. Senate.
[11] Of course, lowering $F_t'(1)$ is a rather simplistic way of describing the consequences of losing confidence in fundamental valuation models. In a way, this loss of confidence means that investors are faced with a sort of more radical kind of uncertainty, called "ambiguity" or "Knightian uncertainty" in the modern literature, where they are not so sure of the model driving the pricing process (see Alan Greenspan's speech in 2005). It has been known since Ellsberg (1961) famous paradox that people do not like ambiguity (for a recent survey see Ilut and Schneider (2023)). In the face of ambiguity, people tend to prepare for the worst-case scenario. Here, it can be argued that the worst-case scenario is that irrational trader have permanently taken control of the market and present-value relations have become useless. Thus, this loss of confidence can have a more dratic impact on the demand function D_t. But lowering $F_t'(1)$ is a simple way to illustrate the main mechanisms at work.

now simply $r_t + \pi_t^f$, the short term interest rates plus what they consider as the normal risk premium for the asset under consideration[12]. What does this mean for asset prices?

The estimated fundamental price P_t^* no longer plays any role since the active fundamentalists have entirely capitulated and the "capitulation" price P_t^c tends to oscillate around the solution of this equation[13]:

$D_t\left(\frac{P_t^c}{E_t(P_{t+H})}, r_t + \pi_t^f\right) = Q_t P_t^c$ (3ter)

To understand what this means, assume again that fundamentalists tend to underestimate the demand for the asset under consideration and overestimate the normal risk premium required by investors. Initially, they were underestimating the fundamental price and were keeping the price undervalued through their short positions. Where does the price P_t^c settle relative to its true fundamental equilibrium once the fundamentalists have capitulated?

Here we have three prices and three expected excess return/apparent risk premia that constitute three extreme kinds of equilibrium depending on the trust placed on fundamentalist analysis.

The price and expected excess return assuming – unrealistically - that active fundamentalists fully control the market (infinite $F_t'(1)$) and force pricing to remain at the (flawed) estimated fair value:

$$D_t\left(\frac{P_t^*}{E_t(P_{t+H})}, r_t + \pi_t^*\right) = Q_t P_t^*$$

The price and expected excess return at the end of the capitulation process.

$$D_t\left(\frac{P_t^c}{E_t(P_{t+H})}, r_t + \pi_t^f\right) = Q_t P_t^c$$

The true fundamental price in an efficient market, under the assumption that fundamentalists are no more victims of private/asymmetric information, understand the underlying dynamic of the market and correct their mistakes (more on that in the last section). In other words, the (unattainable) risk premium and prices that would result from a full-information recursive solution of equation (1ter).

$$D_t\left(\frac{P_t^v}{E_t(P_{t+H})}, r_t + \pi_t^v\right) = Q_t P_t^v$$

The observed price P_t and expected excess return π_t (unobservable, but measured imperfectibly by surveys) will generally be somewhere between these extremes depending on the proportion of passive fundamentalists, the degree of surrender of active fundamentalists (value of $F_t'(1)$, between 0 – full capitulation – and a maximum value – that is not infinite - when investors are highly confident in fundamentalists' estimates) and on the perturbations introduced by the fact that certain investors do not base (rightly or wrongly) their short-term expectations on any type of fundamentalist analysis.

$$D_t\left(\frac{P_t}{E_t(P_{t+H})}, r_t + \pi_t\right) = Q_t P_t$$

[12] Or equivalently, professional investors stop making explicit short-term forecasts and invest in line with their benchmarks. The result is the same since benchmarks have generally been set quite rigidly based on expected normal returns π_t^f.

[13] with fluctuations due to the complex game between chartists and contrarians which we'll discuss later.

With $\pi_t \; = \pi_t^f + +\alpha \, F_t\left(\frac{P_t^*}{P_t}\right) + non-fundamentlist\; perturbations$

In all these relations, the price is a monotonically increasing function of the expected short-term excess return π_t , $\pi_t^*, \pi_t^f, \pi_t^v$. Higher perceived excess return leads to higher demand, and prices must rise to clear the market through two key mechanisms (higher prices mechanically lead to a higher supply $Q_t P_t$ and, hopefully, to a lower "strategic" demand linked to $\frac{P_t}{E_t(P_{t+H})}$).

Thus, the ranking of P_t , P_t^*, P_t^c and P_t^v is the same as for π_t , π_t^*, π_t^f and π_t^v.

The assumption that fundamentalists underestimate the strength of demand means that $\pi_t^* < \pi_t^f$ and $P_t^c > P_t^*$. The price after capitulation is obviously higher than the price that would be observed if the market was under the perfect control of fundamentalists with flawed estimates. We have also $P_t^v > P_t^*$, the true fundamental price is also higher than the fair price badly estimated by fundamentalists.

But what about the capitulation price P_t^c compared to an "efficient" market price, i.e., a price P_t^v resulting from equation (5) with more accurate (or even full information) estimates for current and future risk premia? Because fundamentalists underestimate the strength of the demand, the badly estimated fundamental price P_t^* is lower than any accurate estimate P_t^v of that fundamental price. If $P_t^v > P_t^*$, the underlying risk premium π_t^v is lower than π_t^f. This is a consequence of the fundamental pricing equation (5) used by fundamentalists that establishes a negative relation between risk premia and fair values. And if $\pi_t^f > \pi_t^v$, then $P_t^c > P_t^v$.

This is a very important result. At the end of the capitulation process, asset prices should exceed fair value. This is due to the dual nature of risk premia. When they are supposed to be high, pricing equations used by fundamentalists lead to lower fundamental prices. But when people lose faith in these pricing equations and become passive investors, higher expected "normal" risk premia mean higher expected returns, higher demand, and higher prices.

Thus, when fundamentalists underestimate the strength of demand, we have the following ranking: $\pi_t^f > \pi_t^v > \pi_t^*$ and $P_t^c > P_t^v > P_t^*$. Symmetrically, if they overestimate demand, we'll have $\pi_t^* > \pi_t^v > \pi_t^f$ and $P_t^* > P_t^v > P_t^c$. The current expected return and prices (π_t and P_t) will be close to π_t^* and P_t^* when fundamentalists are highly confident in their estimates and will move towards π_t^f and P_t^c as they capitulate, with unfortunately no built-in mechanism to make them stop at π_t^v and P_t^v.

When the perturbations brought by non-fundamentalists are large, prices and expected returns can be anywhere but continue to obey the following relation:

$$D_t\left(\frac{P_t}{E_t(P_{t+H})}, r_t + \pi_t \right) = Q_t P_t$$

Let's sum up our results. Fundamentalists are always wrong about "normal" risk premia, and they try to stabilize markets at biased prices. Too low if the real equilibrium premia are lower assumed, too high otherwise. If the error is large, they will have a poor investment performance because the market will fail to move in the direction they expect. They can correct their mistake, but they can also capitulate and give less weight to their fundamental estimates. The asset price will then move in the right direction, but it will not miraculously converge towards the true equilibrium price. The process could be chaotic as fundamentalists may lose control of the market and will lead to an overshooting of prices. If the true equilibrium risk premia are lower than the "normal" risk premia assumed by fundamentalists, it is likely that prices will rise even higher than the level warranted by these low risk premia.

Let us make a final remark before turning to the empirical evidence. The price overshoot in the event of fundamentalists' mistakes and capitulations should depend a lot on the characteristics of the asset under consideration. As already noted, when short-term expected returns become disconnected from the fundamental value of the asset, two mechanisms will still equilibrate the market. First, higher/lower prices mechanically lead to higher/lower supply that investors should absorb. This is a powerful but rather crude mechanism. Any absurd expectation about short-term returns can be absorbed into prices through this endogenous supply mechanism. Secondly, if investors are not myopic and also consider long-term expected returns (based on $\frac{P_t}{E_t(P_{t+H})}$), changes in prices continue to affect demand. This second mechanism, which maintains a certain rationality in the behavior of investors and help to balance the markets without extremely absurd prices, is much more powerful for assets whose long-term returns are easier to estimate. This is the case when H is not too large and $E_t(P_{t+H})$ relatively well known. And here, government bonds and equities are at opposite ends of the spectrum (with corporate bonds and exchange rates in the middle). Risk-free government bonds have a fixed redemption value at a not so long horizon (10 years for a 10-year bond), while the equilibrium value for equities is a very difficult to estimate and the convergence process could be very long. When fundamentalists capitulate and short-term expected returns become disconnected from the asset's fundamental value, one can expect a much more violent overshooting process in the equity market than in the government bond market.

2/ Empirical evidence.

In the previous theoretical section, we described how errors by fundamentalists on short-term risk premia could lead to market instability. It is time to review the evidence on the role played by this type of mistakes.

For all markets, there have been episodes of "conundrum" or "irrational exuberance", i.e., periods where fundamentalists have spotted prices at abnormal levels. Even with the benefit of hindsight, for most of these episodes, the academic literature has not been very conclusive about the reasons for these apparent mispricing.

Do we have evidence that prior to some of these episodes, fundamentalists missed some lasting changes in risk premia, produced biased estimates of fundamental values, lost control of markets and contributed to the following overshooting of prices relative to fair value?

We believe that there is strong empirical evidence on these questions in many markets, but we focus in this part our attention on the two key dollar-based markets: the US Treasuries market and the US equity market.

US Treasuries.

US Treasuries are a good place to start for many reasons.

This is a key market where risk-free assets are priced (assuming, as we will do throughout this section, that there is no risk of default and that risk premia are only justified by the duration risk). Pricing of US Treasuries influences pricing in all other markets.

The second reason is that the fundamental pricing relations are particularly simple and easy to manipulate. As said in the previous part, zero-coupon bonds have two very attractive characteristics: they pay no income ($E_t(c_s)$=0) and they have a known final value. Thus, in equation (5), $\sigma_{t,H}^2 = 0$, when

H is the bond's maturity and $P_t^*/E_t(P_{t+H}) = e^{-H R_t^H}$ with R_t^H the zero-coupon rate of maturity H (in its continuous time convention: the traditional actuarial rate is $e^{R_t^H}$-1).

Thus, for zero-coupon bonds, equation (5) is simply:

$$e^{-H R_t^H} = e^{-\int_t^{t+H} E_t(r_s)ds} \; e^{-\int_t^{t+H} E_t(\pi_s^f)ds} \; e^{\int_t^{t+H} \frac{1}{2} E_t(\sigma_s^2)\ ds}$$

Or

$R_t^H = (\int_t^{t+H} E_t(r_s)\, ds + \int_t^{t+H} E_t(\pi_s^f)\, ds - \int_t^{t+H} \frac{1}{2} E_t(\sigma_s^2)\ \ ds)/H$ (6)

In a market controlled by fundamentalists, long-term rates should be an average of expected future short-term rates, plus an average of future expected short-term risk premia, plus a corrective term related to the difference between short-term and long-term risk premia that we discussed before (except for bonds with very long maturities, this corrective term is rather small).

Note that in this expression, the risk premium π_s^f is the risk premium expected on a bond of reduced duration: at date s, the residual maturity of the bond will be H+t-s. Since risk premia depend on the duration of the bonds, we prefer to avoid any ambiguity and write:

$$R_t^H = (\int_t^{t+H} E_t(r_s)\, ds + \int_t^{t+H} E_t(\pi_s^{H+t-s,f})\, ds - \int_t^{t+H} \frac{1}{2} E_t(\sigma_s^2)\ \ ds)/H$$

With $\pi_s^{H,f}$ the risk premium at date s on a bond with maturity H.

Another characteristic of the US Treasuries market is that, due to its central role, we have several available surveys to estimate what are the investors' expectations for both future short rates $E_t(r_s)$ and future long rates $E_t(R_s^H)$ at least for some key horizons s and maturities H. Finally, in an efficient market, there are some strong restrictions on the relationship between risk premia on bonds of different maturities. All bonds are subject to the same risks (monetary policy and risk premia surprises), but with variable intensity depending on their maturity. This variable intensity should be reflected in the relative risk premia (if not, it would be possible to build a leverage portfolio of bonds with no risk but a positive expected return).

All this means that we can extract with a reasonable degree of confidence the trajectory expected by investors for future short-short term rates controlled by central banks ($E_t(r_s)$ for s>t). And therefore, we can have a strong view on the buy-and-hold embedded risk premia estimated by investors. This buy and hold risk premia are $R_t^H - (\int_t^{t+H} E_t(r_s)\, ds)/H$ or $R_t^H - (\int_t^{t+H} E_t(r_s)\, ds - \int_t^{t+H} \frac{1}{2} E_t(\sigma_s^2)\ \ ds)/H$ depending on the chosen convention (geometric or arithmetic average of the future expected excess return). In no other market it is possible to have such an easy estimate of how cheap the market is perceived to be.

More precisely, there are two methods to estimate the expected path for future short rates:

- The survey only approach. Crump et al. (2022) uses the maximum information available in surveys (not only surveys on future short rates, but also on other macroeconomic variables) to understand how people form their expectations on future short rates and extract these expectations (full set of $E_t(r_s)$). They also discuss the embedded buy-and-hold risk premia that result from these estimates.
- The use of "affine term structure models". As we have just said, in efficient markets, risk premia on bonds of various maturities are related to their relative risks. It should not be

possible to build a risk-free portfolio of bonds with a positive guaranteed return. The "surveys only approach" does not guarantee that the produced risk premia comply with these constraints. And when these strong constraints are not respected, it is likely that due to measurement errors the surveys have produced biased estimates of the true short-rates expectations. Thus, it seems very attractive to estimate models where, in addition to surveys results, one introduces as observable variables, the rates R_t^H for different maturities H, plus the relations (6), plus the constraints on relative risk premia due to bonds relative riskiness. The introduction of rates and constraints on risk premia is made by the use of "affine term structure models", a class of model introduced by Duffie and Kan (1996).

The only downside is that this approach assumes some kind of market efficiency (in other words, P_t not too far from P_t^*). When markets are in turmoil and fundamentalists are capitulating, it is not sure that the cross-restrictions on risk premia are still respected. But one of the big advantages of using term structure models is that these models can continue to produce estimates of expected short rates and risk premia even when detailed surveys are not available. Surveys are generally monthly or quarterly (or even annually for some questions), and the deformation of the yield curve between two surveys continues to give an excellent estimate of future expected short rates. The use of models allows the publication of daily estimates and a better real-time understanding of what is happening in this key market.

An interesting question is whether these models make it possible to work without the surveys (and historically the first versions of these models did not use surveys). In other words, is there enough information in the yield curve, given the restrictions on relative risk premia, to extract in normal times the expected path of future short rates without the help of surveys? In some way, this is a very theoretical question: why refuse to use some of the available signals? If some of the surveys are not of a good quality and are useless, the standard statistical procedures that are used will help introduce some large measurement errors and the surveys will not pollute the estimates extracted from the yield curve. Thus, the full use of available surveys seems the right procedure. Now, to answer the theoretical question, two key papers (Kim and Orphanides (2012) and Duffee (2011)) clearly explain why, for two different reasons – the "small sample" and "hidden factors" problems - using only the yield curve is likely to produce rather poor estimates. We briefly discuss this point, because there is a fundamental implication to this observation: one cannot hope to overcome the market failures due to private information on current risk premia using only observed prices and sophisticated statistical models.

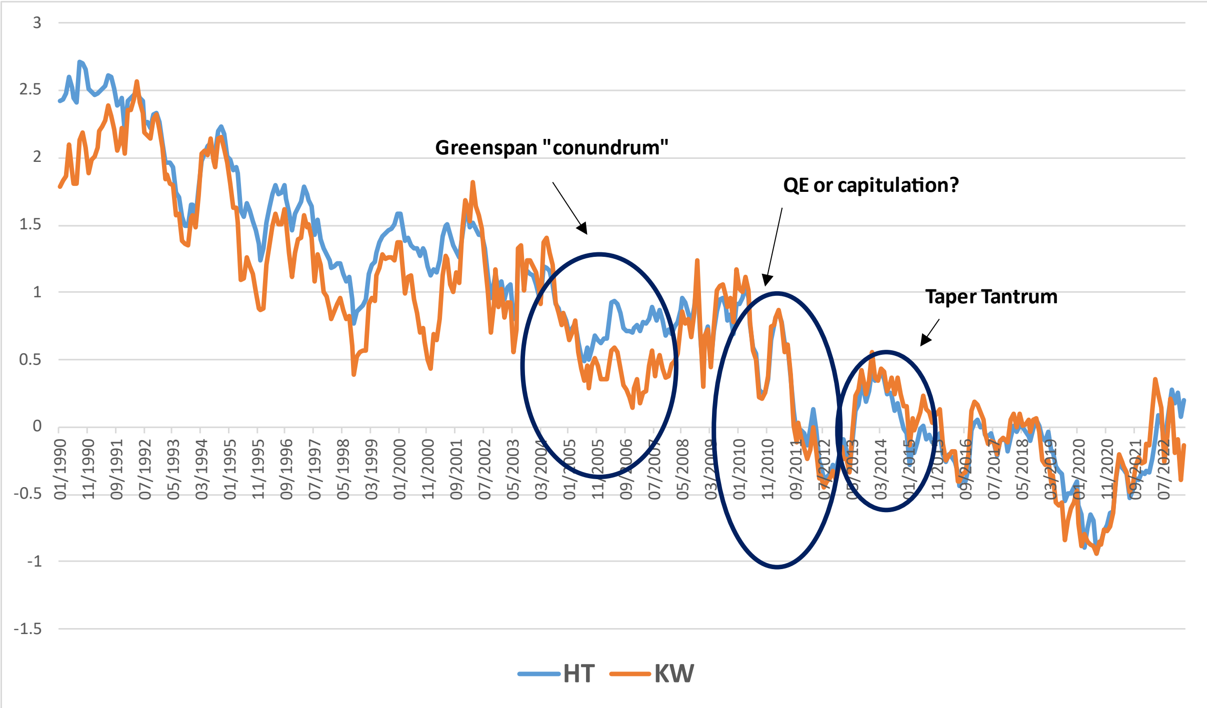
Here we compare two available estimates produced by yield curve models using surveys as one of the inputs.

- The KW estimate, based on Kim and Wright (2005). Daily data is available on the Fed's website[14].
- The HT estimate based on Hördahl and Tristani (2014). These estimates are not available on the BIS website, but Peter Hördahl kindly provided us with the monthly data.

In the following chart, we have plotted the two estimated contributions of risk premia to 10-year rates since the early 1990.

[14] https://www.federalreserve.gov/data/three-factor-nominal-term-structure-model.htm

Chart 1: contribution of risk premia to 10-year US Treasury rates.

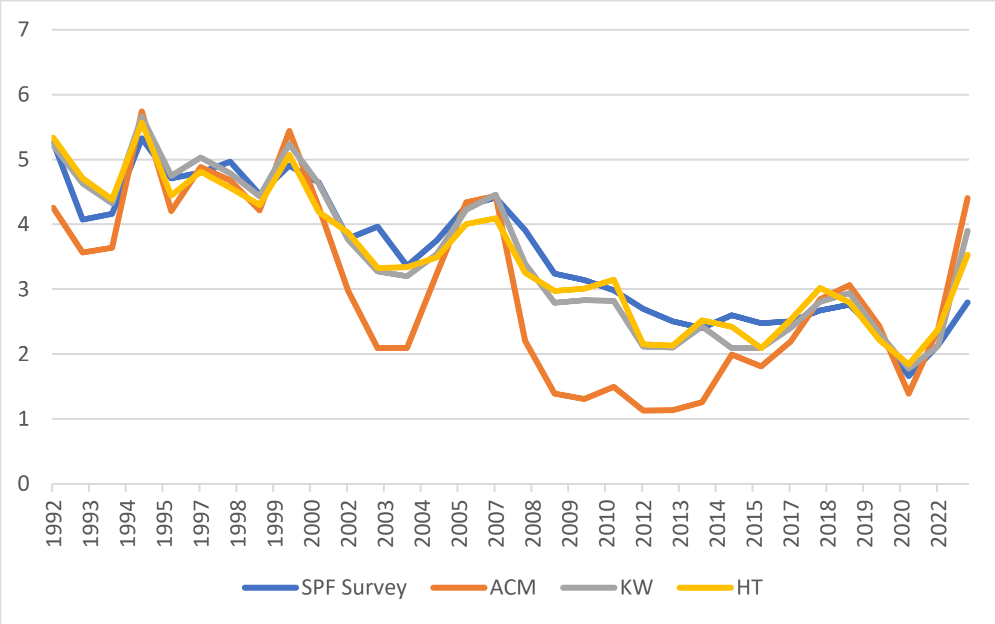


These two estimates are usually very close, and the likely reason is that expectations measured by surveys play a major role in both estimates[15]. This approach is effectively a "survey plus approach" in the sense that since any observed yield curve is compatible with many credible scenarios of short rates and risk premia (the "hidden factor" problem explained by Duffee (2011)), surveys play a central role in the estimates. To show the strong anchoring provided by the surveys, the following chart compares four estimates of the average short rates expected over the coming 10 years: the raw estimate taken directly from the Survey of Professional Forecasters, the KW and HT estimates, and the estimate coming from another well-known model (ACM) that relies solely on the observed yield curve[16].

[15] Although they don't use the same sources: HT uses the Federal Reserve Bank of Philadelphia's quarterly Survey of Professional Forecasters (SPF) while KW uses the Blue Chip Financial Forecasts. But the results of these surveys are generally very similar.

[16] Adrian et al. (2013). Daily ACM estimates are available on the New-York Fed website at https://www.newyorkfed.org/research/data_indicators/term-premia-tabs#/interactive.

Chart 2: Average expected short rates over the coming 10 years according to the SPF survey, KW, HT and ACM (in February of each year, when the 10-year SPF forecast is available).



This chart shows that KW and HT are generally close to the survey results. These complex affine-term term structure models do not necessarily add much value when detailed surveys on expectations are available, but as already mentioned, one of their key benefits is to allow the use of the yield curve to extrapolate expectations between two exhaustive surveys.

What we see in HT and KW estimates is a rather chaotic process driving embedded risk premia from around 2.5% in the early 1990s (i.e., 10-year rates 250 basis points above the short rates expected on average in the next 10 years) to a low close to -1% in the summer of 2020, before a rebound in positive territory in 2022 (and 2023 according to HT). Over these three decades, there have been fundamental changes in the balance between supply and demand for US Treasuries. Numerus papers have pointed to various structural reasons for this long-term trend (see for example Kim and White (2005) who attempted to explain Greenspan's conundrum in real time). The reasons can be separated into two categories: reasons linked to a reduction in the risks borne by investors (notably lower inflation risks thanks to a more credible Fed) and reasons linked to a better balance between the supply and demand of bonds (with higher demand perhaps due to demographics and rising central banks' reserves, and, later, a lower supply due quantitative easing (QE)). But, while some reasonable explanations have been identified ex post, this downward structural trend has been rather chaotic. There have been two periods of accelerated decline (in the early 2000s and 2010s) and periods of extremely volatile risk premia (for example in 2013 at the time of the "tapper tantrum" and in 2020 during the COVID-19 crisis). We believe that the chaotic nature of this process is partly due to the mistakes made by fundamentalists and the variable control they have exercised in this key market.

To make the link with our theoretical part, these "buy-and-hold" risk premia embedded into the market ($\int_t^{t+10} E_t(\pi_s^{10+t-s})\, ds/10$) must be compared to the short-term excess returns expected by investors $\pi_t^{H,f}$, for bonds of various maturities H.

There are several reputable surveys that measure investors' short-term expectations of future long-term rates and many academic papers have used these surveys to extract the short-term returns expected by investors and the short-term risk premia that they required (see for example Piazzesi et al. (2015) and Nagel and Xu (2023)).

Here, we estimate one-year risk premia on 10-year Treasuries based on the response to the monthly "Consensus Economics" (CE) survey that began at in late 1989. Obviously, only a subset of investors responds to this survey (mostly economists), so we are not getting exactly the true risk premia required by the average investor. We'll return to this issue while discussing the results.

The one-year risk premia are estimated by comparing the 10-year rate expected in one year with the 10-year forward rate at the same horizon priced into the yield curve. If the forward rate is higher than the expected rate, you can buy the 10-year bond forward at an attractive rate and expect to make a profit by selling this bond in one year at a lower rate. The CE survey asks questions about the future 10-year par rate with the traditional US semi-annual coupon convention. Thus, we have to extract from the yield curve the forward rate on the same basis. Fortunately, based on the work of Gürkaynak et al. (2007), the New-York Fed publishes on its website estimates of the zero-coupon yield curve with all the parameters necessary for calculating forward rates at all horizons for zero-coupon rates as for par rates with a semi-annual coupon convention.

Once the difference at date t between the forward rates priced into the yield curve and the 10-year rates expected in one year $(F^{10}(t,1) - E_t(R_{t+1}^{10}))$ is determined, the next step is to measure the risk premium expected on this bond.

Theoretically, it suffices to multiply the difference by the bond's modified duration. But to be completely rigorous, two corrections should be made. First, there is the traditional problem of convexity. Suppose investors see the risk as symmetric around their central forecast. An unexpected drop in rates of 50 basis points will produce an unexpected gain of 4.94% (for rates at 4%) while an unexpected increase of 50 basis points will result in a loss of 4.68%. There is a difference of 0.25%. Thus, due to this convexity phenomenon, there is a small positive expected return even when the expected rate is equal to the forward rate. This small correction could be estimated using the historical volatility of bonds prices, but as it is generally done in the literature, we do not introduce here this correction. Second, buying a 10-year bond forward is exactly the same as buying now a 11-year bond funded by a 1-year debt. Thus, the collected risk premium is not the risk premium on a 10-year bond rolled over one year, but the risk premium on a 11-year bond minus the risk premium on a 1-year bond hold over a one-year period. Davanne (2021) shows that the risk premium on US Treasuries is not strictly proportional to duration and that in general the risk premium on short-term Treasuries has been higher than warranted by their duration. Thus, the risk premium on a 11-year bond minus the risk premium on a 1-year bond is generally somewhat lower than the risk premium on a 10-year bond. Again, the difference between the forward rate and the expected rate slightly underestimates the true risk premia required on 10-year Treasuries. Like most of the literature, we forget the need for this small duration correction and consider in what follows that the risk premia are proportional to durations. Thus, the risk premium required by investors on 10-year Treasuries could be approximated by $(F^{10}(t,1) - E_t(r_{t+1}^{10}))$ multiplied by the 10-year Treasuries' modified duration. For a 10-year zero-coupon, assuming again than risk premia are (almost) proportional to duration, we have to correct for the difference of duration between a par bond and a zero-coupon bond, and we get the following key approximation[17]:

$Pr^{10}(t) = 10/(1 + E_t(r_{t+1}^{10}))\ (F^{10}(t,1) - E_t(r_{t+1}^{10}))$ (7)

With $Pr^{10}(t)$ the average risk premium over the coming year to hold on a rolling basis a 10-year zero-coupon Treasury, $E_t(r_{t+1}^{10})$ the 10-year par rate expected in one year according to the CE survey, $F^{10}(t,1)$ the forward 10-year par rate priced on-year ahead into the yield curve according to the Fed's data.

However, we introduce in (7) another important type of correction that we have not seen in the literature. There are two sources of noise that can be avoided in the estimate issued from (7). First, we don't know exactly at what time people responded to the survey and what the exact level of forward rates was. In volatile markets, forward rates can change quickly. Second, the forward rates published by the Fed are extracted from off-the-run Treasuries while people answer to surveys about benchmark rates, i.e., on-the-run easy to trade securities. The bias can be quite large in period of market instability. Indeed, in some papers, one can see large risk premia on US Treasuries at the height of the 2008-2009 financial crisis, as the low expected on-the-run rates over the one-year horizon are compared to high current off-the-run rates.

One way to eliminate these two sources of noise is to estimate the risk premia expected by investors on a horizon between 3 months and 1 year (and not between today and 1 year). This expected return over a period of 9 months is given by the following relation, which is the profit expected over the full one-year period minus the profit expected for the first three months:

$10/(1 + E_t(r_{t+1}^{10}))\ (F^{10}(t,1) - E_t(r_{t+1}^{10}))$- $10/(1 + E_t(r_{t+0.25}^{10}))\ (F^{10}(t,0.25) - E_t(r_{t+0.25}^{10}))$

As the modified duration is very close considered at t+1 or t+0.25, we can simplify this expression as

$$10/(1 + E_t(r_{t+1}^{10}))\ (F^{10}(t,1) - E_t(r_{t+1}^{10}) - \ (F^{10}(t,0.25) - E_t(r_{t+0.25}^{10}))$$

$$10/(1 + E_t(r_{t+1}^{10}))\ ((F^{10}(t,1) - F^{10}(t,0.25)) - (E_t(r_{t+1}^{10}) - E_t(r_{t+0.25}^{10})))$$

The two significant sources of noise that we have just discussed impact the two forward rates ($F^{10}(t,1)\ and\ F^{10}(t,0{,}25)$) in a very similar manner. Thus, this estimated risk premium expected at the horizon between 3 months and one year that depends on the difference between these two forward rates will be measured much more precisely than the risk premium over the full one-year horizon.

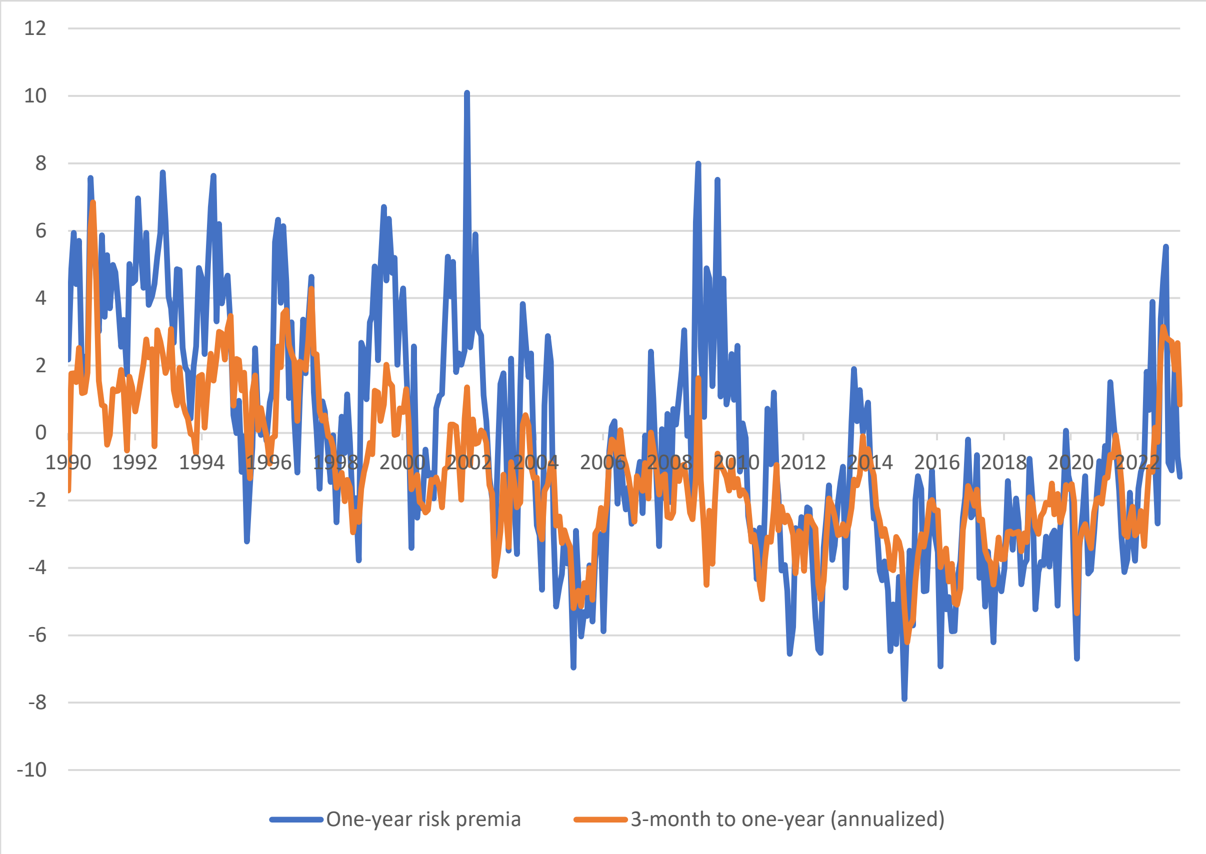
This risk premium expected over this 9-month period can be annualized to give:

$$Pr^{*\,10}(t) = \frac{40}{3\ (1+E_t(r_{t+1}^{10}))}\ \left[\ F^{10}(t,1) - F^{10}(t,0.25) - (E_t(r_{t+1}^{10}) - E_t(r_{t+0.25}^{10}))\right] \qquad (2)$$

The following chart shows the two risk premia: the traditional one - $Pr^{10}(t)$ - consistent with what is generally done in the literature and the (annualized) "clean" risk premium estimated at the horizon between 3 months and one year - $Pr^{*\,10}(t)$.

[17] As $E_t(r_{t+1}^{10})$ is the expected par rate and not the expected zero-coupon rate, $10/(1 + E_t(r_{t+1}^{10}))$ is not exactly the modified duration of the zero-coupon rate. But the difference is very small. Other approximations are much more important.

Chart 3: Short term risk premia on 10-year Treasuries.



As expected, the estimated risk premia at a horizon between 3 months and one year is much less volatile than the risk premia estimated at the one-year horizon. But they have a rather similar profile over the last 30 years. They were on average significantly positive until the late 1990s, then started to decline. They reached extremely negative levels in 2005 (on average -4.1% that year for the 3-month to one-year risk premia), picked up a little in the following years while remaining negative and then renewed with on average very negative levels in the last decade until the recent inflation crisis. In the fall of 2022, risk premia reached levels not seen since the late 1990s.

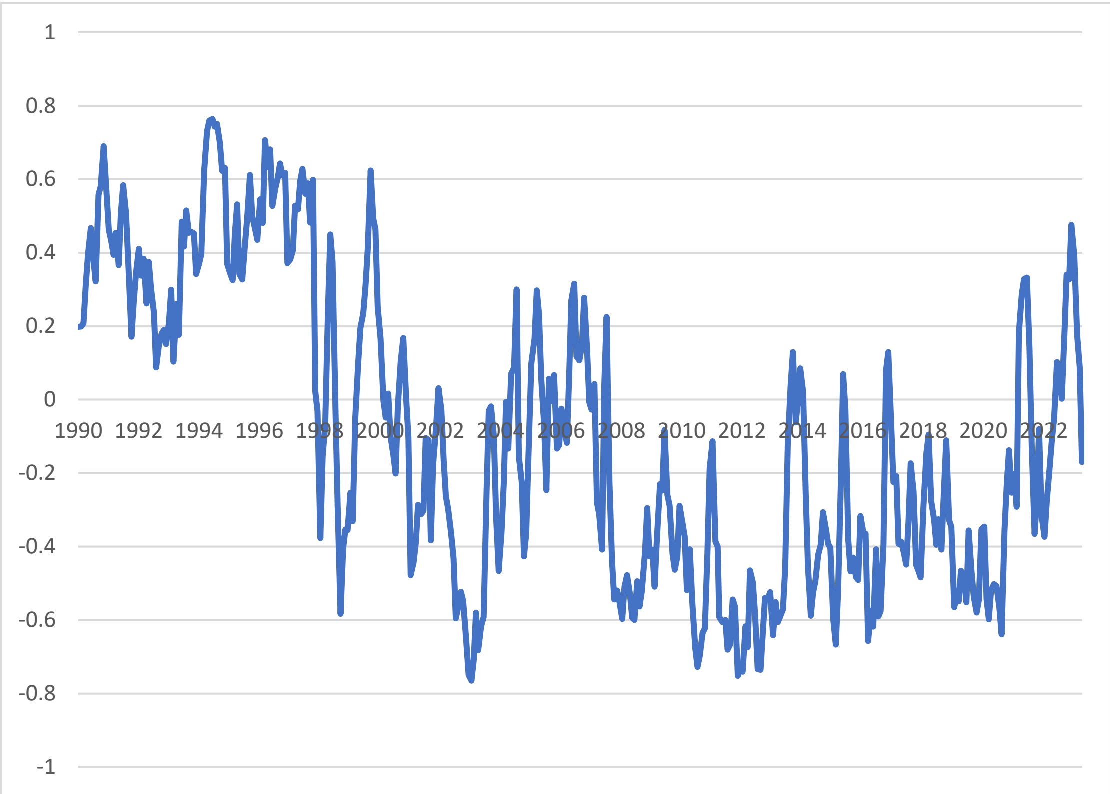
We are going to use these "clean" short-term risk premia to better understand the downward trend in risk premia embedded in long-term Treasuries (chart 1). But first we have to check that what we have done does not produce pure noise. Can we make sense of these long-term low frequency trends, and of the high short-term volatility that can be seen in chart 3?

We believe that these estimates show the importance of two key determinants of the short-term risk premia required on US Treasuries.

First, the correlation between the price of US Treasuries and the price of other risky assets. This is a major result of the Capital Asset Pricing Model that risk premia on specific asset classes should not be related to volatility per se, but to the "betas", where correlations play a major role. A very volatile asset with a negative correlation to other risky assets should provide a large negative risk premium since investors can use it as a hedging instrument.

It is well known that there was a structural reversal in the correlation between bonds and equities in the late 1990s as shown in chart 4.

Chart 4: Correlation between the price of 10-year Treasuries and the S&P500 index (daily data and rolling periods of 3 months).



Many papers (see for example Cieslak and Pang (2021) or Campbell et al. (2020)) explain that this correlation depends very much on the nature of the shocks impacting the economy. Some shocks create a positive correlation (higher inflation generally lowers bond and stock prices), while others (higher productivity, financial crisis) have the opposite impact. The changing nature of risks since the mid-1990s (with a more credible Fed and more financial instability) probably explains the structural change in correlations.

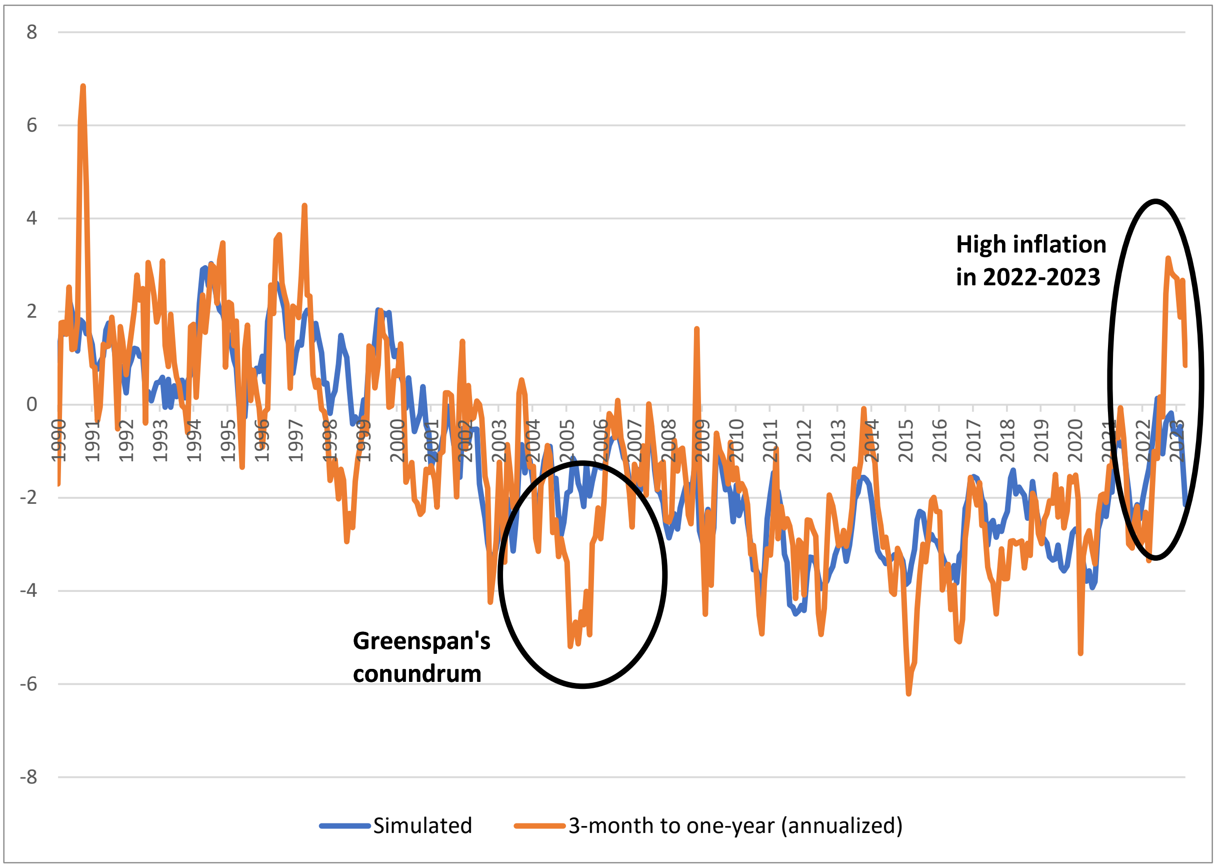
Changes in the correlation between Treasuries and other risky assets may explain both low-frequency and high frequency changes in risk premia. The current correlation likely plays a role in how much excess return investors require. Thus, the high-frequency volatility seen in chart 4 should explain part of the high-frequency volatility observed in chart 3. But the correlation observed on average over several recent years probably also plays a role because it influences investors' strategic allocations. For example, the fact that Treasuries play a useful hedging role in the event of a financial crisis will encourage a strong position in Treasuries, even if for cyclical reasons, the current correlation is apparently broadly neutral. In other words, both instantaneous correlations and average correlations observed in recent years probably play a role.

Another factor seems to play a major role in explaining not the observed trends but the short-term volatility apparent in chart 3. When the recent performance of Treasuries has been bad, for example since the beginning of 2022, investors seem to require higher risk premia. This may be explained by the constraints that professional investors face. They often have a set risk budget and if they lose money on a position, they may be forced to close that position, unless the short-term expected excess return improves.

In other words, we would expect three variables to have some explanatory power to understand the "clean" risk premia that we have estimated. There is obviously the recent correlation between 10-year Treasuries and the S&P500 index (observed correlation during the last 3-months, variable COR), but also the average correlation over the last 5 years (COR5), and the gains or losses of a buying position in 10-year Treasuries in the last 6 months (difference between the observed rate and the forward rate priced into the yield curve six months ago, variable SURP).

These three variables are indeed very significant, and they explain quite well the evolution of risk premia over the last 30 years. When the correlation switches durably from 0.5 to -0.5, the risk premium ultimately declines by almost 5% (from very positive to very negative). If long term rates have risen unexpectedly by 100 basis points over the last 6 months, the risk premium rise by almost 1%. The following chart compares the risk premia extracted from surveys and the risk premia simulated with the help of these three simple variables[18].

Chart 5: Short-term risk premia on 10-year US Treasuries (observed and simulated).



The only periods with a lasting gap between the observed risk premium and its simulated value were at the time of the Greenspan's conundrum where economists were likely losing control of the market (and their expectations were probably no longer representative of the global consensus), and recently,

[18] Here is the equation estimated over the period January 1990 – April 2023, with students into brackets.

$Pr^{*\,10}$ = -0.62 + 1.33 COR + 3.49 COR5 + 0.89 SURP + 0.73 *AR(1)

(-3.77) (4.30) (6.56) (7.68) (21.03)

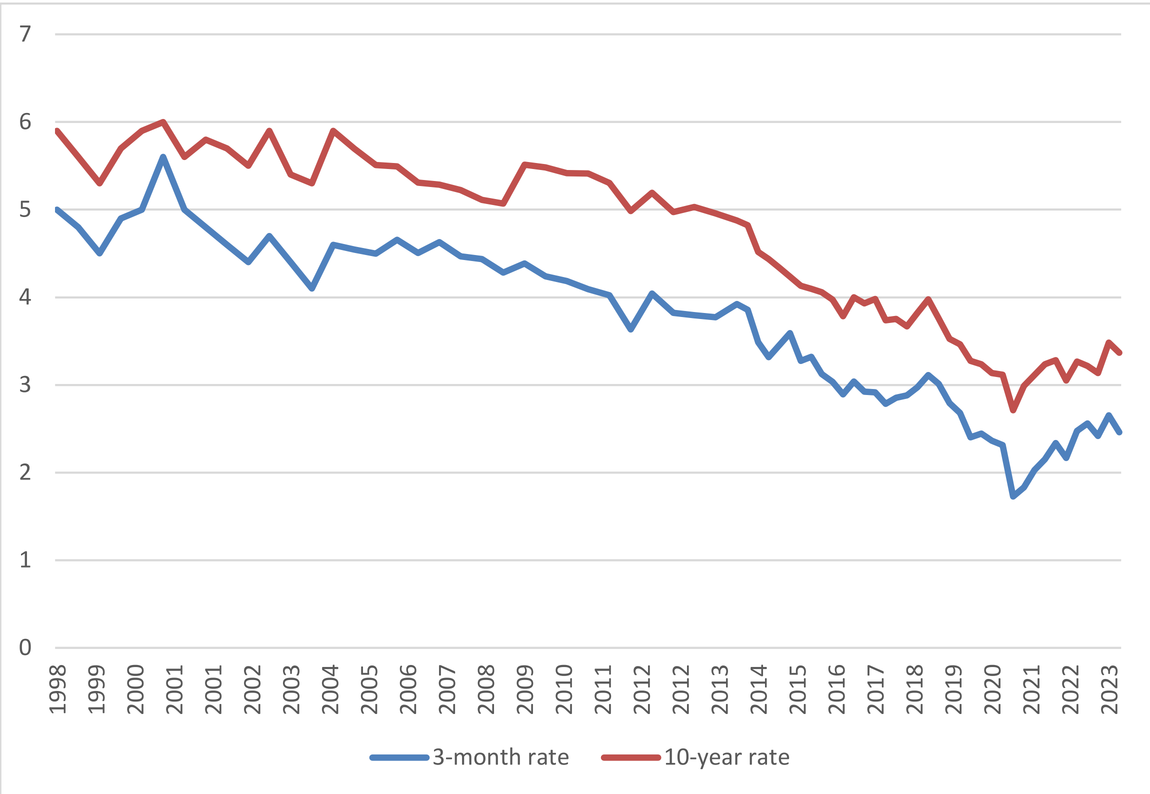
$R^2 = 0.84$. Monthly residuals are auto correlated (ρ=0.73). The simulation in chart 5 is based only on the three key variables, without the auto correlation effect.

since mid-2022, with a surge in the rate of inflation and a sharp increase in observed risk premia that our simple equation could not reproduce.

The next step is to ask whether fundamentalists have been able to spot the available signals in the late 1990s and early 2000s, and quickly adjust their fair value estimates. The answer is clearly no. The common view among market practitioners – but not academic economists - has always been and still is that a normal yield curve is upward sloping. Despite the fact that the basic CAPM clearly proves that betas, not volatility, should drive risk premia, there is still the widespread view that a long-term bond is riskier than a monetary investment due to its price volatility, even for bonds issuers with negligible risk of default. An inverted yield curve is seen by most as an anomaly, and indeed a good predictor of an incoming recession.

A good way to illustrate this widespread view of "normal" risk premia is to look at the long-term forecasts produced by economists as measured by "Consensus Economics" (semi-annual question from April 1998 to April 2014, and quarterly since). At the 6-10 year horizon, expected 10-year rates are always significantly higher than expected 3-month rates, and the expected spread has not changed much since the late 1990s despite the structural break in short-term risk premia.

Chart 6: 3-month and 10-year rates expected at the 6-10 years horizon.



In other words, fundamentalists still believe that in a normal situation, $(\int_0^{10} \pi_s^{10-s,f}\, ds)/10 \approx 1\%$.

Let's assume again as an approximation that normal risk premia are proportional to durations, $(\pi^{10-s,f} = \frac{10-s}{10}\pi^{10,f})$. The previous relation means that economists still believe that in the long term, the normal short term risk premium on 10-year bonds is still close to 2%, as in the 1990s.

Further evidence that fundamentalists failed to spot the structural break in bonds risk premia in the late 1990s is the way they reacted to rates diverging from estimates of fair value. Over the last 20-years, long-term rates have often surprised observers as too low. It has often been difficult to understand the behavior of this key market. As mentioned earlier, in 2005 Federal Reserve Chairman Alan Greenspan famously spoke of a "bond conundrum" mentioning the possibility that bond prices could be a short-term aberration. In September 2018, Cohen et al. (2018) also noted in the BIS quarterly bulletin that "*In recent years, government bond yields have not always responded predictably to macroeconomic or monetary policy news. Long-term yields in the United States remained stubbornly low even as the Federal Reserve initiated a series of interest rate hikes away from zero starting in late 2015*". But they did not explain this new conundrum.

As shown in the first section, this difference between the observed rates and the rates considered normal is a logical consequence of erroneous estimates of the required risk premia. But these discrepancies have always been treated as "conundrum", or as the temporary consequences of Quantitative Easing (more on that later), and never led to a fundamental reassessment of how fundamentalists analyze the pricing of US Treasuries.

What were the consequences for the US Treasuries market of these lasting mistakes? Did we see as expected the loss of confidence in fundamentalist forecasts, the capitulation of fundamentalist investors, an increase in volatility and a clear overshooting of US Treasuries prices compared to what can be considered, with the benefit of insight, like fair pricing?

We are not going to describe precisely here how this market has behaved over the last 25 years (see Davanne (2021) for a more detailed analysis[19]). But it is clear that very often we have seen signs of fundamentalists starting to lose the confidence of market and higher volatility as a result. As already mentioned, this was the case in 2004-2005 (Greenspan's "conundrum") or in 2018 (the BIS's conundrum…). Several times long-term US Treasuries rallied unexpectedly (inflicting large losses to some hedge funds betting on higher yields on a fundamental basis).

Yet, it does not seem that we can observe the entire story with full capitulation of fundamentalist investors. Rates on long-term US Treasuries were often puzzling, but never at absurdly extreme levels (as was the case with the US equities, which we will examine shortly). Indeed, something seems to be missing if we compare the behavior of the US Treasuries market and what our theoretical part implicitly predicted. With full capitulation, short term expected return should converge to the "normal" risk premium expected by investors ($\pi_t = \pi_t^f$, here around 2%). In other words, with complete capitulation, the "consensus economics" survey should have shown a rebound in expected excess returns at some point. This was not the case.

There are several possible explanations. Firstly, perhaps most investors capitulated and stopped believing in fundamentalist models, but the economists interviewed in this survey were the last men standing with expected excess returns no more representative of the expectations of the average investors. Secondly, maybe, the US Treasury market is special. As we mentioned, the final/redemption value is perfectly known. Thus, the average long-term return is also perfectly known, and this may prohibit complete capitulation because when long term rates are extremely low, investors understand that the short-term return cannot be very high.

[19] In this paper, we use an original affine term structure model with more factors than in KW and HT to describe the expected path of short-term risk premia more accurately. The estimation of these new factors is made possible using surveys on future long-term rates, and therefore new information on expected returns on long-term bonds at different horizons. In other words, we can extract much more accurately not only the average path ($\int_0^{10} \pi_s^{10-s,f}\, ds$), but also the expected risk premia $\pi_s^{H,f}$ for all horizons and all maturities.

Both explanations likely played a role, but we believe that there is also another factor that prevented a full capitulation: fundamentalists were twice lucky.

First, their bearish bets at the time of the Greenspan's conundrum ultimately paid off despite their pricing errors. This was because the economy performed better than expected between 2005 and the financial crisis that began in 2007. Short-term rates rose much more than expected, pushing long-term rates higher. The short positions taken by fundamentalists were right for the wrong reasons, which delayed the day of reckoning. In the following years, 2008-2009, the financial crisis triggered so much volatility in short rates expectations that the mispricing signals (long-term rates and past returns not at their expected levels) were too weak to challenge the fundamentalist forecasts and feed new "conundrum" speeches.

Second, we would argue that the introduction of Quantitative Easing starting in 2010 saved the fundamentalists and delayed judgement day again. When US long-term rates began to decline to extremely low levels fundamentalists had not expected, there was an easy culprit: the market distortions that the Fed's massive buying had created. Thus, their credibility was not too affected. Moreover, fundamentalists began to realize that short-term risk premia were abnormally low and they indeed improved their models. All the literature – academic and professional – emphasized that the transmission channel of QE passed through the fall in risk premia. Thus, QE forced fundamentalist analysts to revisit their estimates – explicit or implicit - of current risk premia ($\pi_t^{H,f}$) and the risk premia in the not-too-distant future ($E_t(\pi_s^{H,f}$) for dates s close to t). In other words, the gap between the true short term expected return (π_t^H) and the normal current excess return expected by fundamentalist ($\pi_t^{H,f}$) narrowed, and as explained in the first section, this is this gap that produces puzzling phenomenon that destroy the fundamentalists' credibility. Moreover, more recently, the spike in inflation rates in 2021-2023 has pushed up required short-term risk premia and also helped structurally bearish fundamentalists preserve their credibility.

But this is not a stable equilibrium. Fundamentalist investors still believe that normal risk premia have not changed much over the last 30 years (see again chart 6), and they seem to overestimate the role of the relatively recent QE in the structural decline in short term risk premia. More important is probably the credibility of central banks in the fight against inflation. If the Fed succeeds in bringing inflation back close to its 2% target, short-term risk premia may stabilize around the very low (negative) levels seen over the past 25 years. This could create numerous new conundrums in the future.

US equities.

Let's start by identifying the most interesting periods of abnormal pricing in the US equity market.

Unfortunately, in the equity market, we don't have the equivalent of KW or HT, sophisticated models able to extract what is priced into the equity market (future profits at the firm and aggregated level and risk premia) from the available surveys and observed prices.

To the best of our knowledge, the closest approach is that of Polk et al. (2006) where the restrictions on risk premia imposed by the CAPM were used to extract the market risk premium from the observed stock prices of a panel of companies. But with no surveys used and an overreliance on the CAPM as risk premia theory, the extracted risk premia were not very precise, and their model does not seem to be used. More recently, an interesting new literature has emerged that attempts to extract from observed prices, in particular futures on dividends, the changes in risk premia and cash flows expectations (see for example Koijen and Gormsen (2020) for a price-based analysis of how investors changed their expectations following the outbreak of COVID-19). But the focus is usually on understanding the sources of market volatility, rather than extracting the level of the underlying variables (a much more challenging task) [20] .

As a second-best to assess the risk premia embedded in current prices, a good starting point is currently the excess CAPE yield developed by Nobel Laureate Robert Shiller. The excess CAPE yield is the difference between the earning yield (Earnings/price) and the real long-term interest rates. In Shiller's estimates, the earning yield is smoothed using average earnings over the past 10 years to limit the impact of the business cycle (abnormally low earnings during recessions). The real long-term interest rate is the US Treasury 10-year rate minus the average inflation rate over the past 10 years.

The excess CAPE yield is not exactly a measure of the market-embedded risk premium (the term $e^{\int_t^{t+H} E_t(\pi_s)ds}$ in equation (5)). It is a static measure of risk premia that does not take into account the fact that earnings will grow over time as the size of the economy increases. However, as shown in the following chart updated regularly by Robert Shiller[21], the excess CAPE yield has been a remarkable predictor of future long-term excess returns for equities[22].

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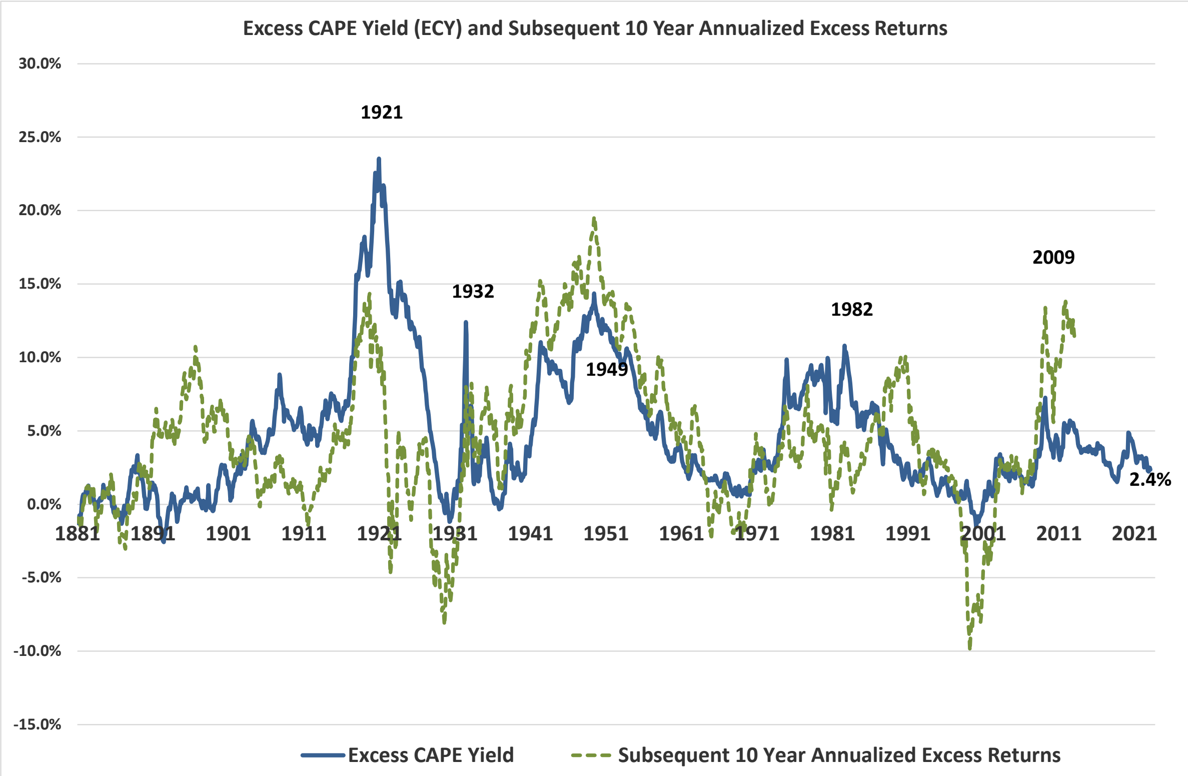
[20] The specificities of the US Treasuries market (simplicity of the fundamental valuation model, easy linearization in continuous time, quality of available surveys) are probably the reasons why this market seems to be far ahead of the stock market in this type of complex modelling. But there is no reason to believe that the general approach used in affine term structure models could not be applied to the stock market. Equation (5) is a bit more complicated for equities than for bonds (with the introduction of the dividend yield and no well-defined terminal price), but the information needed to extract risk premia seems even more abundant for equities. When we have a few available maturities for bonds (and basically only three factors to look at, the level, the slope and the curvature of the yield curve), we have the prices for hundreds of large-cap stocks of various risks (plus some interesting futures on dividends) and historical data on analysts' expectations for earnings, sales and dividends at different horizons.

[21] http://www.econ.yale.edu/~shiller/data.htm, last accessed 6 April 2023.

[22] Note also that economic growth does not automatically increase the expected return on equities.Higher future earnings are funded by current earnings reinvested in tangible and intangible assets, and less cash is available to shareholders today through dividends or share buybacks. Growth is unambiguously beneficial to shareholders only when companies benefit from some form of rent and enjoy the benefit of growth without having to pay a fair price.

Chart 7: Excess CAPE yield for the S&P500 index.

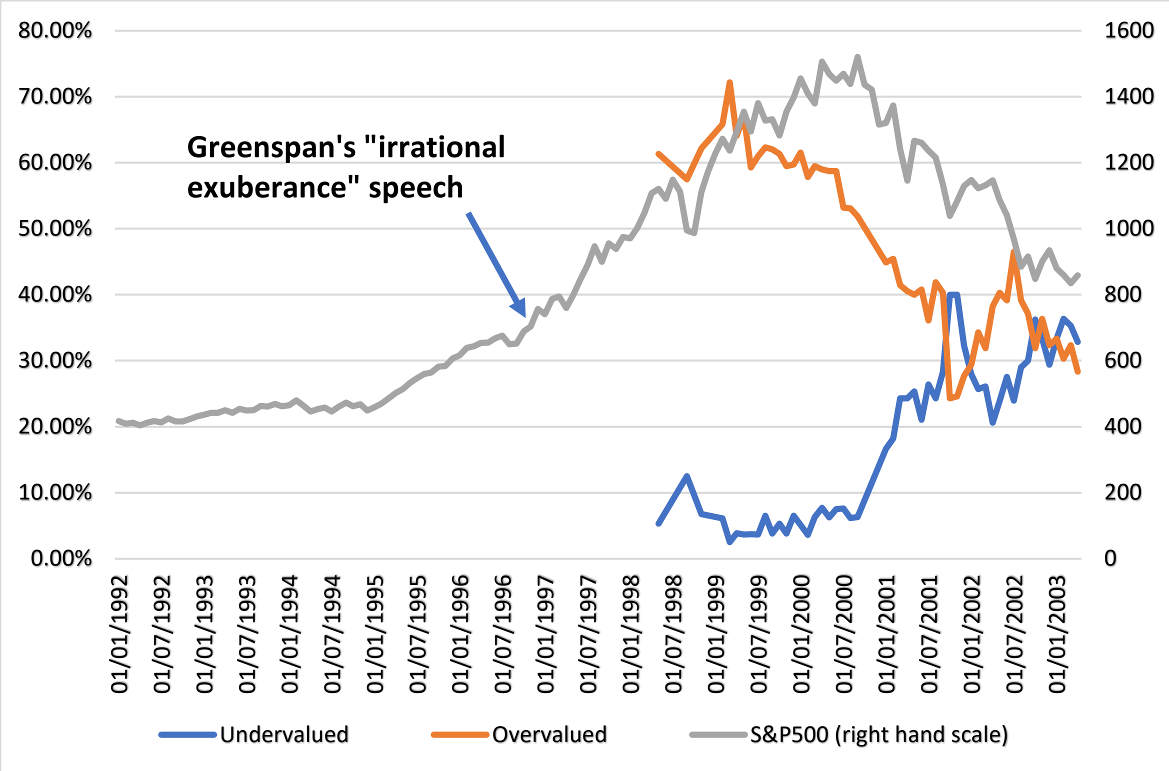


In this chart, the most abnormal period since the Second World War was the years 1999-2000. The excess CAPE Yield was then negative for 19 months, bottoming out at -1.5% in January 2000. Not surprisingly, the 10-year annualized excess return that followed was particularly poor.

There is little doubt that the US equity market was then extremely overvalued. What is interesting is that the diagnosis was already firmly established at that time. Alan Greenspan famously spoke of "irrational exuberance" as early as December 1996. Moreover, during this period, Gallup conducted (mostly) monthly surveys to gauge how individual investors active in the market rated investment opportunities. The UBS/Gallup survey relied on a large panel (1,000) of individual investors with over 10,000 dollars invested in financial assets. Among other questions, they were asked whether the US stock market was overvalued, valued about right or undervalued.

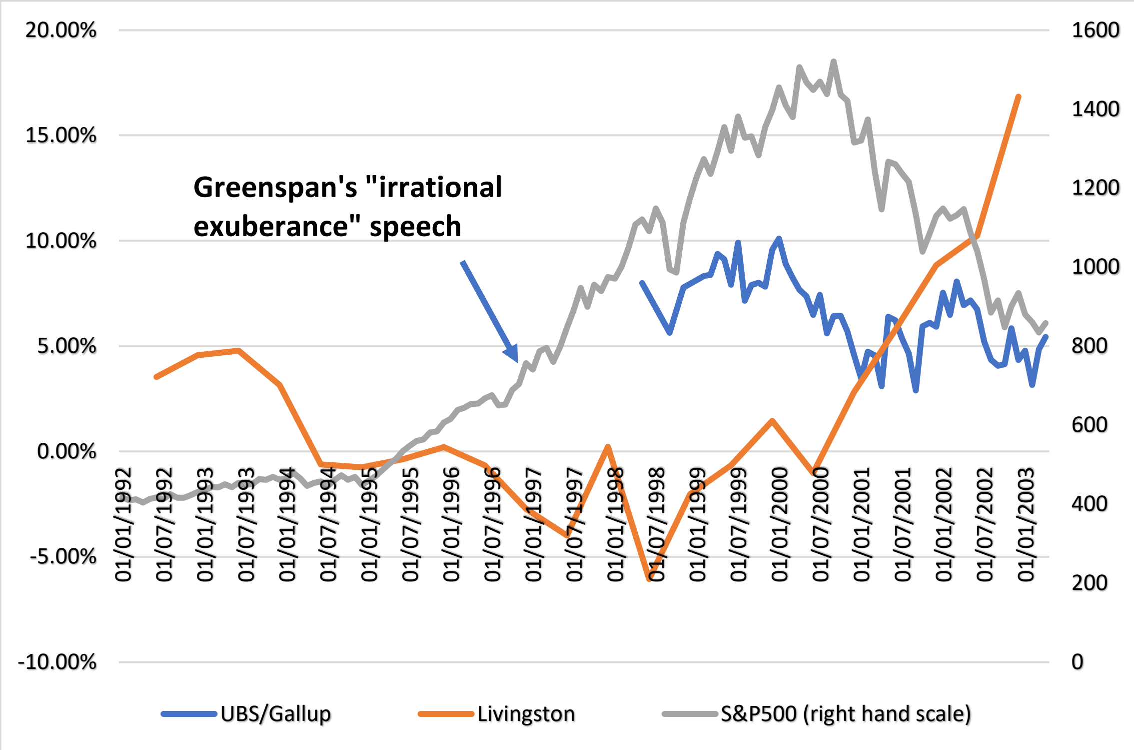
Among investors expressing an opinion (generally around 80% of the panel), a large majority thought in the late 1990s that the market was overvalued (see chart). A minority thought it was about right (probably believing that some of the high prices were justified by the coming internet revolution) and almost no investors thought that the market was undervalued.

Chart 8: Overvaluation of the US stock market at the end of the 1990s according to the UBS/Gallup survey.



This was a clear case of fundamentalists capitulation, with most investors showing the right diagnosis (the market is overvalued) and keeping large long positions. With $P_t \gg P_t^*$, expected future returns $\left(r_t + \pi_t + F_t\left(\frac{P_t^*}{P_t}\right)\right)$ should have been low, but they were not. The UBS/Gallup also asked individual investors about their expectations for stocks returns in the forthcoming twelve months. In the following chart, we give the implicit excess return/risk premium extracted from these expectations (expected return minus one-year risk-free rate). We have also added the same expected excess return extracted from another survey, the bi-annual Livingstone survey managed by the Federal Reserve Bank of Philadelphia.

Chart 9: One-year risk premia according to surveys (left hand scale) and S&P500 index.



Both categories of investors seem to have capitulated and reduced the weight given to fundamental analysis, but not to the same extent. At the peak of the first half of 2000, individual investors were expecting excess returns on average of 8.1%, probably close to what they considered as the normal equity risk premium at that time. In the two Livingstone surveys carried out at the peak of the market (December 1999 and June 2000), the professionals questioned were much less enthusiastic than individual investors, but they were not frankly pessimistic (expected excess return of 1.5% in December 1999 and -1% in June 2000). Respondents to the Livingstone survey clearly reduced the weight given to fundamentalist analysis: they were much less pessimistic about equities future returns at the top than they had been in the previous years at much lower valuations. Yet, unlike individual investors, they did not fully capitulate. Note, however, that the Livingstone survey questions professional economists and not professional fund managers. It is easier to stick (partially) to a fundamentalist view of the world when you are not an actual fund manager who is evaluated and rewarded based on his investment performance.

Thus, to varying degrees depending on the type of investors, there was a massive capitulation of fundamentalists in the second half of the 1990s. The only question is why?

It is tempting to see in this the consequence of the sheer irrationality of certain investors. In this "explanation", there was a self-fulfilling bubble with a rising market attracting even more irrational investors fearful of missing out on an opportunity to get rich easily, thanks in particular to the forthcoming internet revolution. But there is another possible story strongly supported by the Livingston survey and other evidence (including Alan Greenspan's "irrational exuberance" speech early in the capitulation process). It is a story of fundamentalists – including Alan Greenspan – getting their risk premia assumptions wrong and gradually losing control of the stock market. Starting in June 1994, the excess return expected in the Livingstone survey began to be systematically negative (see chart 9). In the three years before Alan Greenspan's irrational exuberance speech, the average short-term risk premium was -0.8% (six surveys from 1994 to 1996).

The normal annual risk premium expected by fundamentalists was probably above 5% (the historical average stock market outperformance over a one-year period since 1962 is 6.6%[23]). Using the conventions of our theoretical part, $\pi_t^* \approx -0.8\%$ while $\pi_t^f > +5\%$ during this period. This discrepancy means fundamentalists have likely gotten the fair value massively wrong and could explain why they lost control of the stock market.

It is important to note that a negative π_t^* before many investors capitulated does not mean that the true equilibrium risk premium (π_t^v in our theoretical part) was also negative. As discussed, when fundamentalists underestimate the strength of demand, we have $\pi_t^f > \pi_t^v > \pi_t^*$. The fundamentalists keep the price at an abnormally low level for a while and therefore the supply of the risky asset ($Q_t P_t$ in equation (1)) is also abnormally low. With less private information and better fundamentalists, prices would have been higher, and the short-term excess return expected by investors would also have been higher.

This is an important point. Surveys on expectations are very useful to spot mispricing. Indeed, they sent a powerful signal in the mid-90s that fundamentalists investors were missing something. But when markets are mispriced, they don't directly and easily lead to the "true" risk premia that should be used in fundamentalists' asset pricing.

Why were US equity risk premia ($\pi_t^v, \pi_t \;\; or \; \pi_t^*$) low in the mid-1990s?

There are at least five important reasons why the "market" risk premium (i.e., the average short-term or tactical risk premium on all listed stocks) may vary:

- Of course, the risks vary. Certain periods are characterized by particularly significant financial risks (for example during a run on the financial system as in 2008-2009) or geopolitical risks. During these periods, investors will demand higher than normal returns to hold the supply of stocks.
- The ability to bear financial risks is also very dependent on the level of wealth. Indeed, many products offer certain guarantees on the invested capital. When wealth declines, for example during a recession, many investors need to reduce the level of risk in their portfolios.
- The level of interest rates also seems to play a large role in the demand for risky assets. Extremely low interest rates appear to induce investors to take on more risk to increase expected returns (i.e. they accept a lower risk premium). This is the famous TINA argument ("There Is No Alternative") put forward in 2020-2021 to explain the large flows into expensive stocks. This dependence of risk premia on the level of interest rates does not seem very consistent with the traditional utility functions of rational risk adverse investors. Low interest rates mean that investors expected future wealth should be lower, and generally it is considered that lower wealth should lead to less risky investments. In fact, the role of extremely low interest rates is best understood in the context of behavioral finance and the "prospect theory" (Kahneman and Tversky (1979)). An important discovery of behavioral finance is the key role of "loss aversion". People do not like to lose money, even a small amount. Thus, they can accept the risk of large losses with relatively low probability to avoid the near certainty of small losses. Buying expensive stocks can be dangerous but leaves open

[23] For the S&P500 index. As we discussed in the theoretical part, due to the negative autocorrelation of returns, the long-term excess return over this 61 years period was much lower than one would (naively) expect from a 6.6% (arithmetic) average. Over this entire period, the average annual excess return on a geometric basis was 4.9%.

the possibility of being on the winning side, while investing in safe assets when real interest rates are negative is associated with the certainty of a loss.

- Public interventions in financial markets also have an impact on risk premia. Historically, the "too-big-to-fail" policy has led governments to assume the losses in the event of major shocks that deplete the capital of financial institutions (banks, insurance companies, pension funds…). When governments tighten regulations and introduce "bail-in" instruments to protect the taxpayer, it should lead to higher risk premia in the equity market.
- Risk premia are influenced by financial innovations (for better or for worse…) and fads in investment techniques. Financial institutions are very creative to package risky assets in such a way as to make them more attractive (until they are no longer so…). For example, many observers fear that the current success of funds specialized in unlisted "private" assets may be due to the lack of mark-to-market pricing of these assets. This allows fund managers to smooth the return on their funds and to some extent hide the true riskiness of their products.

Thus, for all these reasons – linked to fundamental and/or psychological changes - the strength of demand for stocks varies over time and the short-term/tactical risk premium that equilibrates the market also varies, sometimes for many years.

In the second half of the 1990s, interest rates were not particularly low, and the powerful "TINA" mechanism probably did not drive the strong demand for equities. But there were at least two phenomena that boosted the demand for risky assets at the time. First, it was a period of exceptional macroeconomic stability with the absence of recession between 1991 and 2001. This macroeconomic stability was attributed both to the skill of the Fed, chaired since 1987 by Alan Greenspan, and to a more stable "new economy" based on services and intangible assets. Thus, earnings seem more stable and investing in equities less risky. As far as investment techniques are concerned, it was the era of the cult of equities "always winning in the long term". Specifically, many investors, including pension funds, massively increased the share of equities in their strategic allocations. For defined benefit pensions funds, this was a rather naïve approach to risks which will be very costly and will be followed in the next decade by a U-turn – encouraged by new regulations - and a better balance between assets and liabilities.

All of this led to a structurally very strong demand for equities. Using the formalization of the theoretical part, the function $D_t\left(\frac{P_t}{E_t(P_{t+H})}, R_t\right)$ probably shifted upwards[24]. The market found its equilibrium only thanks to the fall in short-term risk premia (and later in the decade, after the fundamentalists' capitulation, an overshooting of equity prices such that the strong demand for equities could be satisfied by the endogenous increase in supply $Q_t P_t$).

[24] Another key aspect of this period was also the rigidity of strategic asset allocations. For many (most?) investors, the long-term expected return was not estimated and adjusted based on $\frac{P_t}{E_t(P_{t+H})}$, but using what they considered as the normal risk premium (again π_t in our formalization). This undermined one of the key mechanisms that allow prices to balance supply and demand.

3/ The future of asset pricing.

Of course, fundamentalist investors cannot avoid making mistakes. In this paper, we have stressed that there are two very different types of "mistakes". First, the future is inherently uncertain - this includes monetary policy, profits, the supply of assets, and so on. Everyone knows this and active fundamentalist investors do their best to incorporate incoming information into their forecasts.

There is another type of mistake, more subtle and dangerous, which is not due to the uncertain nature of the future, but to private information: fundamentalist investors may be wrong about the strength of the current underlying demand for risky assets and use inaccurate risk premia assumption in their fair value estimates. We have shown, both theoretically and empirically, that such mistakes can lead to large mispricing and market instability.

An important observation is that these errors help to understand the success of certain non-fundamentalist investment techniques. Errors on risk premia create trends in asset prices. It can be a massive multi-year trend, as seen in the equity or bond markets when big mistakes go uncorrected, and fundamentalists capitulate. Or a shorter trend when mistakes don't last long or are gradually corrected. In other words, in the way markets work today, the learning process about changing risk premia seem very inefficient and only months or years of conundrum may (or not) convince fundamentalists to correct their pricing mistakes.

In this inefficient learning process, non-fundamentalist investors play their part:

- "Momentum" or "chartist" investors believe in the existence of trends (and they are often right). So, when the fundamentalists start to capitulate, they will amplify the process and in some way help send prices in the right direction. But they will also likely contribute to the overshooting process, as they are not "better fundamentalists" and have no ways of identifying when the trend is starting to create another kind of mispricing (with our notations, when traveling from P_t^* to P_t^c, P_t unfortunately crosses P_t^v).
- "Contrarian investors" also profit from the mistakes of fundamentalists. Contrarian investors take positions opposite to the consensus (and they are also often right). If the expected excess returns, measured through surveys or direct contacts with other market participants, appear abnormally low ($\pi_t^f > \pi_t$), they will buy the asset rather than sell it like other investors. And like the chartists, they will first send prices in the right direction. But like chartists, they have no way of identifying when to stop buying. When the asset price reaches fair value (P_t^v), expected excess return still looks a bit low ($\pi_t^f > \pi_t^v$), and contrarians can keep pushing prices higher. Yet in this process, contrarians appear to be far less dangerous than momentum investors. As prices get closer to their "capitulation peak" (P_t^c), they have progressively less incentives to keep buying (as π_t converges towards π_t^f). They can even fight against the worst excesses of trend followers if they try to send prices above the "capitulation peak" (P_t^c). In this case, the expected excess returns become larger than "normal" ($\pi_t > \pi_t^f$) and contrarians become sellers of the asset.

The result of this complex game between the fundamentalists making mistakes, momentum investors and contrarians is a very inefficient learning process when the underlying balance between supply and demand (i.e., π_t^v) is changing. The long-term "objective" risk premia embedded in markets become quite volatile (see chart 1) as markets try to find their equilibrium.

This inefficient learning process is probably extremely costly from a welfare point of view. Thus, a fundamental question is why fundamentalists seem unable to correct their mistakes before capitulating? In other words, why Alan Greenspan said in 2005 that "Bond price movements may be a short-term aberration, but it will be some time before we are able to better judge the forces underlying recent experience", introducing some form of "ambiguity" or "Knightian uncertainty" in the mind of fundamentalists rather than "there has been a structural break in the correlation between bonds and equities, which justifies the sharp drop in risk premia on US Treasuries that has been apparent for some time in available surveys"?

There is no obvious answer to this question. But there is a clear factor that has contributed to this situation: the difficulty to use expectations surveys to improve works on valuation. We have emphasized that the information we can get from these surveys depend very much on how healthy markets are, i.e., depends both on the control of fundamentalists and the accuracy of their fair value estimates. Using the notations of our theoretical part, we are looking for π_t^v and the surveys give π_t which can be anywhere between π_t^* and π_t^f.

Moreover, existing surveys have many drawbacks to estimate the current excess returns π_t expected by investors.

- As we have said on several occasions, there is often a problem of investors' representativeness. Most surveys with a long-term history (for example, the SPF and Livingston Surveys managed by the Philadelphia Fed, the "consensus economics" survey) focus on professional economists rather than actual investors. Their primary objective is not to measure financial risk premia, but rather to know what professional economists expect and these surveys often focus on macroeconomic forecasts (this is the case with the SPF and the "consensus economics" surveys) with financial market forecasts a sort of secondary by-product (for example, there are unfortunately no forecasts about equity markets in the "Consensus economics" survey). In normal times, when the market is well controlled by fundamentalists, it is likely that the excess returns expected by these professional economists are a good approximation of the excess returns expected by the real-world investing community. But, when the market situation becomes puzzling, investors might capitulate faster than economists, which can result in these surveys providing biased estimates of the true average expected excess returns.
- Since these surveys are not designed to measure risk premia, the questions do not relate to expected returns, but to future prices (equity indices, interest rates, exchange rates). There is therefore work to be done to translate the answers into estimates of risk premia. This is not always easy (see our discussion of how we construct "clean" estimates of US Treasuries risk premia from the responses to "consensus economics"). A common problem is that we don't know exactly where the prices were when people gave their forecasts. This introduces a lot of noise, and we may need to extract the expected return between two future dates (three months and one year ahead forecasts in the case of the answers to "consensus economics"). But, in doing so, we are missing out on the very short-term expected returns that can play an important role in some investors' decisions.
- Last but not least, an important point that we have not mentioned yet is that people are likely to respond to these surveys by giving their most likely expectation. But market-embedded risk premia are based on average expected returns, not the most likely expected return. If extreme risks are asymmetric, the most likely expected return will give a biased estimated of the average expected return that plays a key role in investors' decisions (see the discussion of this point in Diercks and Jendoubi (2023)).

If fundamentalists finally understood the key role that expectations surveys should play in their estimates, it is likely that the organizations running the surveys would try to facilitate the estimation of risk premia (perhaps by asking questions directly on expected returns, with also questions on the distribution of these returns and not only the central scenario). It is also likely that more representative surveys would emerge or re-emerge (as noted, the UBS/Gallup questions on expected future returns covered 1,000 individual investors between 1998 and 2003, with very interesting results).

Public authorities should encourage this movement. For instance, we should acknowledge the Federal Reserve Bank of Philadelphia which twice in the past saved very useful surveys which were progressively dying due to the declining interest of the sponsoring organizations[25]. In theory, good surveys could remove, or at least reduce, the drastic difference in positions of short-term risk premia relative to other fundamental factors in valuation work. There is in principle a deep symmetry between the roles played by future payoffs, short-term interest rates and short-term risk premia in asset prices (see again equation (5)). The current fair value depends on the expected path of these three key variables in a very similar way. However, the radical difference lies in the fact that while the starting points for pay off and short-term rates are known, the same is not true for current short-term risk premia. Well-designed representative surveys are needed to give equal status to all three variables, and remove the dangerous vulnerabilities created by the unobservability of short-term risk premia.

In addition, the current situation creates an asymmetrical information situation that would deserve some studies. Investors at the center of the financial system (trading desks of investment banks, hedge funds) have a much better view than anyone of how the market finds its equilibrium. Informally, they do their own polls by talking to other investors. Even if they don't incorporate this knowledge into explicit estimates of fair value, one wonders if their strong historical trading results aren't at least partly due to this informational advantage. It is never good to have this kind of asymmetric information situation which can negatively impact market liquidity and provide unwarranted rents. Better surveys can also have a positive impact as they would promote a more efficient and equitable distribution of information.

Thus, we would advocate in this paper an optimistic view about the future of asset pricing: the weakness in the way fundamentalists operate could be corrected with better surveys and a good understanding on how to use them. In the words of Shiller, the weak skills of fundamentalist debaters could be vastly improved[26]…

But it is important to understand that this would not suppress all the volatility due to changing risk premia, but only the most extreme overshooting linked to fundamentalists' mistakes and capitulation.

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[25] The SPF survey began in 1968 and was initially conducted by the American Statistical Association and the National Bureau of Economic Research. The Federal Reserve Bank of Philadelphia took over the survey in 1990. The Livingston Survey was started in 1946 by the late columnist Joseph Livingston. The Federal Reserve Bank of Philadelphia also took responsibility for the survey in 1990.

[26] This is a complex point that we'll not develop in this paper, but it would also help that all active fundamentalists better understand the relation (5) and the key role played by $\int_t^{t+H} E_t(\pi_s^f)\, ds$ in the estimated fair value for all asset classes (or equivalently, for equities, its discrete-time approximation proposed by Campbell and Shiller (1988)). Many fundamentalists are probably not fully aware that the long-term risk premia they use should be an average of expected short-term risk premia. A naïve and superficial reading of the literature based on the SDF may give the sentiment that it is possible to jump directly to a series of long-term risk premia based on the SDF. With the SDF, the present value relations for equities can be written directly as $P_t = E_t \sum_{j=t}^{+\infty} m_{t,t+j}\, d_t$ (see Cochrane (2005), page 24). This looks rather simple and attractive, but there is no way to estimate the SDF $m_{t,t+j}$ at long-term horizons, and the correct approach is, as in Campbell and Shiller (1988) or equation (5), to discount future pay-off d_t with the sum of expected future short-term risk premia.

Better fundamentalists would realize that $\int_t^{t+H} E_t(\pi_s^v)\,ds$ is as important as $\int_t^{t+H} E_t(r_s\ )\,ds$ for correctly discounting future cash flows. Starting from better estimates of the current π_s^v , they would cautiously estimate the likely path of future short-term risk premia. Currently, most of the attention of fundamentalists is devoted to monetary policy and cash-flows, and discussions around risk premia should become much more important.

But even with more information on the starting point (π_t^v deducted from π_t), building a robust path for future short-term risk premia $E_t(\pi_s^v)$ would stay very challenging for two key reasons:

- Due to the limitations of the available surveys, we have little reliable long-term data on the evolution over time of short-term risk premia on most asset classes (as already discussed, the US Treasuries market is an exception with several surveys available, albeit with some difficulties in extracting "clean" risk premia). With little data, it is not surprising that we do not have clear explanatory models that could be used to predict future short-term risk premia.
- Another key aspect of short-term risk premia is that they follow long-lasting trends. All the structural factors that influence demand and supply usually change gradually over long periods of time (changes in correlations, in the perception of risk, in the available financial products, in the supply of government bonds, etc..).

The underlying statistical process looks like:

$$\pi_{t+1}^v = \pi_t^v + \mu_t + \varepsilon_t$$

$$\mu_t = (1-\rho)\,\bar{\mu} + \rho\,\mu_{t-1} + \nu_t$$

With μ_t the persistent trend of the short-term risk premium following an AR(1) process due to the ongoing structural changes in the balance between supply and demand, ε_t some short-term noise (or measurement error if we are speaking of the surveys' results), ρ determining the degree of persistence of the trend and ν_t the shocks modifying the trend.

Obviously, the resulting fair value is extremely dependent on the current trend μ_t, since future expected short-term risk premia should have a strong influence on current prices. For example, with $\rho = 1$ (persistent trend) and H=10, in continuous time $\int_t^{t+10} E_t(\pi_s^v)\,ds = H\pi_t^v v + 50\,\mu_t$

If the risk premium increases by 0.5% each year over 10 years (admittedly quite an extreme scenario), the current fair price is 25% lower than it would be with the risk premium unchanged[27].

The bottom line is that changing trends in risk premia can produce a lot of rational volatility in prices, while the lack of clear theory could leave investors dependent on pure statistical methods to extract the trend from noisy survey results. Thus, to the irreducible volatility due to the inflections of the trends, will be added the noise due to the difficulty of measuring the trend in real time.

Of course, the key role of changing trends in market volatility is not specific to short-term risk premia, and it also plays a role for cash-flows. Timmermans (1993) explained how, with an uncertain trend in dividends, learning can produce some volatility, while Pastor and Veronese (2009) also showed how

[27] This is only an illustrative calculus to show the strength of the mechanism. But an important point that we will not discuss here is that the payoffs rationally expected in the future should not be independent of the risk premia assumptions. At the long-term equilibrium, changing risk premia impact all other fundamental variables (short-term interest rates, profits and dividends…). This should somewhat lower the impact of a change in expected future risk premia on current fair values.

the natural volatility due to changing trends in dividends can be strongly increased by the learning process when the signals are noisy.

This literature is very relevant to understand the impact that changing trends in risk premia should have on market volatility. When it comes to short-term rates and earnings, the signals are probably less noisy because fundamentalists can use diverse and rich theories about macroeconomics, the reaction function of central banks, or the determinants of corporate margins. These theories can be tested using years of data on these key variables. Thus, the recent behavior of these variables is only one of many inputs used in an attempt to forecast the future. Fundamentalists' forecasts are changed as new information emerges, but they have little reason to overreact to this new information[28]. But for short-term risk premia, as we said, there are little long-term data and underlying theories. Thus, identifying current trends is likely to be much more difficult than for other variables.

This discussion of the role of trends in asset pricing is important. The high volatility of asset prices can be seen as the price to pay to reconciliate two major apparently contradictory observations.

- On the one hand, fundamental variables are subject to trends. This is particularly true for risk premia. Demand and supply are not stable and are impacted by strong medium-term or long-term trends. On the demand side, we have mentioned many factors – from changing correlations to "TINA" or new regulations – which are gradually changing investors' asset allocations. On the supply side, the accumulation of debts over time – public debt or foreign debt – gradually modifies the short-term risk premia necessary for market equilibrium (think for example of the future consequences of Quantitative Tightening). In the specific case of the US Treasuries market, we saw the impact of these gradual structural changes and the negative trend for the short-term risk premia in the late 1990s and early 2000s (see chart 5). Thus, one could expect asset prices to show long-lasting trends with rising prices when demand tends to grow more rapidly than supply, or declining prices in the opposite case.
- But on the other hand, stable trends disconnected from risks cannot exist in financial markets. Once they are visible, they attract arbitrageurs that make them disappear (or become much more complex). In efficient financial markets, prices should react to new information by jumping from the old equilibrium to the new one. Thus, the trend will be sufficiently perturbed by this succession of jumps that it is no more exploitable by any form of simple statistical learning. In other words, thanks to the work of arbitrageurs and this succession of jumps, there may still be some apparent predictability ex post, but no more obvious predictability ex ante (or in-sample) that could enrich the most basic trend followers.

But this unavoidable volatility around existing trends can have two origins. Ideally, it should come from rational fundamentalist arbitrageurs doing a good job of making the markets jump when the news indicates a change in trend. Or it can be imposed by a complex game between different types of non-fundamentalist arbitrageurs (quants, trend followers, contrarians…) trying to exploit the mistakes of fundamentalists.

[28] Indeed, they may even underreact. This was the conclusion of Piazzesi et al.(2015): "survey forecasts of interest rates are made as if both the level and the slope of the yield curve are more persistent than under common statistical models".

In other words, again, there won't be any miracles: the inevitable volatility due to shifting risk premia won't go away quickly, with active fundamentalists doing a much better job, with the help of better surveys, in their search for fair values. But we can hope that the radical price overshooting due to the capitulation of the fundamentalists would be less frequent. Also, hopefully, volatility could settle into a long-term downtrend as active fundamentalists gain more experience and years of data from better surveys accumulate.

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